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VERTICAL AND HORIZONTAL PROCESSES IN PROBLEM SOLVING [1]

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The present paper is concerned with *an* approach—and not *the* approach—to the universally appealing but nevertheless unpopular research area of problem solving. Problems of problem solving have proved to be particularly refractory to psychologists. More often than not the uncommon researcher with the temerity to attack some aspect of reasoning retreats to more secure and conventional problems when he discovers that his sorties fail to achieve any impressive victory. As a result the literature of problem solving is almost chaotic because it is so heavily sprinkled with isolated bits of information (Duncan, 1959).

Perhaps the present stage of development of psychology does not justify the strategy of investigating such a complex phenomenon. Fortunately, or not, science has no built-in traffic lights to inform investigators when to proceed. It may be a risky and potentially unfruitful gambit to investigate problem solving but then again it may not be. In addition to the intrinsic interest of the area it does offer a challenge to those psychologists who are interested in testing the generality of any set of theoretical principles stemming from other areas of behavior (e.g., learning, perception).

This paper initially will make fleeting references to some methodological problems with which a researcher in the field of reasoning must contend. Then a simple pretheoretical model of problem solving will be described, followed by a report of research which the model generated, and which in turn is shaping the model itself.

METHODOLOGICAL PROBLEMS IN PROBLEM SOLVING RESEARCH

Anybody who does research is—or should be—aware that every decision he makes cannot be justified by facts or logic. Some decisions must be made on the basis of personal intuition. This is particularly true for the researcher in problem solving who must make three strategic decisions which cannot help but have profound influences on his research and the ideas they generate (Kendler, 1961). These decisions, which are not completely independent, are related to the place of problem solving in psychology, the use of complex or simple experimental

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tasks, and the selection of a pretheoretical model to guide research. Considering the volitional nature of these problems, as well as the current status of psychological knowledge, it would be both inappropriate and erroneous to consider these methodological problems as offering only one sensible alternative. Adopting this point of view would do much to minimize the needless disputation that seems to perennially surround matters of research strategy.

Accepting the principle that a basic research strategy is not simply an outgrowth of logical and factual considerations does not reduce one to making decisions in either a haphazard or random manner. A given strategy can be adopted on the basis of rational considerations as long as it is realized that other reasonable attitudes might lead to the adoption of different decisions.

The history of problem solving in particular and psychology in general suggests that problem solving can best be conceptualized not as a basic psychological process, but instead as one that reflects the interaction of more fundamental processes (e.g., learning, perception, and motivation).

If problem solving is not viewed as a unitary process, how is an appropriate experimental situation selected to investigate it? One possibility is that a problem can be selected from a "true life" situation such as troubleshooting electronic equipment. Or problems can be invented (Duncker, 1945; Maier, 1930) that capture the flavor, if only partially, of problems we meet in everyday life.

A more analytical approach can be taken to the selection of an experimental situation to investigate problem solving. If problem solving is compounded of elementary behavioral processes, then it may be more strategic to devise some simple problems in which the relationships of fundamental psychological mechanisms to problem solving are highlighted. That is, tasks should be devised not to duplicate or imitate everyday problems, but instead to isolate and magnify the basic mechanisms that operate in such complex tasks.

This analytical approach which is favored by the authors suffers from one major drawback. How is it possible to know the basic mechanisms of problem solving prior to their discovery? Obviously, excepting divination, there is no method. But this does not prevent the analytical approach from operating. The researcher can prejudge theoretical issues by formulating a model of what he guesses problem solving to be like. The model can guide the investigator in selecting the hypotheses to test, as well as the experimental situations in which to test them.

This brings us to the third and most important decision a problem solving researcher has to make: his choice of a pretheoretical model (Koch, 1959). A pretheoretical model is not equivalent to a theory. The criterion of validity cannot properly be applied to it because essentially a pretheoretical model is an informal conception that operates as an analogy (Lachman, 1960). It is conceivable that different models (e.g., learning, perception, information theory) can all lead to fruitful and valid theories of problem solving.

Psychologists have many possibilities from which to choose their model. These models can be conveniently divided into two main categories: the empirical model that springs primarily from experimental data, and the formal model that is usually generated by mathematical or logical systems. Among the empirical models that have achieved some acceptance are those

that are based on introspective findings (e.g., the four successive stage model of "preparation," "incubation," "inspiration," and finally "verification"), the facts of perception, and those of learning. Some formal models used are those dependent upon stochastic models, game theory, and the operation of computers.

The present authors adopted an S-R learning pretheoretical model. The decision no doubt was influenced by professional training and past research efforts. But other considerations entered. For the past 4 decades S-R learning psychologists have probably been the most active experimental and theoretical group in psychology. To some, if not a large, extent this can be attributed to the fruitful and cleansing effect S-R language has upon designing, reporting, and interpreting research. S-R language forces the psychologist to focus his attention on objectively defined environmental and behavior variables and thus encourages the collection of data and the testing of ideas. The efforts of S-R learning psychologists have supplied a host of facts, concepts, and hypotheses that can be exploited in an exploratory excursion into the realm of problem solving.

The facts and theories of learning, however, do not spontaneously coalesce to form a model that can guide research in problem solving. Some selection must be made. S-R learning theory does not represent a single organized formulation. Anyone who is familiar with the systematic orientations of Hull (1952), Guthrie (1952), Spence (1956), and Skinner (1953) is aware of this. Many of these systematic differences, however, become attenuated and some even disappear when viewed from the distance of problem solving behavior. It is possible and perhaps even profitable to develop a learning model for problem solving that ignores many of the points of disagreement among S-R theories.

Much of the objection to S-R language stems from the apparent discrepancy between active, flowing behavior and the inert, static, single S-R association. Using S-R language does not mean that complex behavior *actually* consists of S-R connections. After analyzing the concept of light, Toulmin (1953), concludes: "We do not *find* light atomized into individual rays: we *represent* it as consisting of such rays" (p. 29). Applying the same idea to the concept of the S-R association: "We do not *find* behavior atomized into individual S-R associations: we *represent* it as consisting of such S-R associations." The concept of the S-R association, therefore, must be judged not in terms of its ability to provide a clear image of behavior, but rather in its capacity to represent the facts of behavior.

PRETHEORETICAL MODEL OF PROBLEM SOLVING

An S-R model needs to represent two important characteristics of problem solving behavior. These characteristics are behavior is continuous, and at any one time behavior consists of several habits. The terms "horizontal" and "vertical" are used to refer to these processes; horizontal to the continuity of behavior against the dimension of time, and vertical to the assumption that independent levels of behavior (i.e., S-R units) occur simultaneously.

The assumption that S-R associations do not occur in isolation, but instead are linked together to form integrated, continuous behavior goes back many years (e.g., Watson, 1913). Today the process is most commonly referred to as chaining. Skinner (1953) and his associates have developed powerful techniques that shape

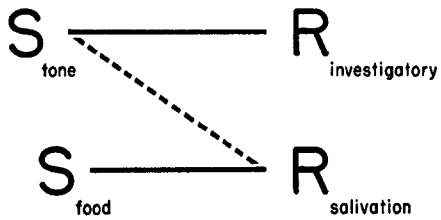

FIG. 1. An S-R representation of classical conditioning.

behavior into long, complicated chains. The mass of data they have collected suggests important principles governing habit chaining. There is litle doubt that when their quasitheoretical system is exploited fully with autoinstructional devices that important insights into problem solving behavior will emerge, particularly in relation to how an added bit of knowledge can trigger problem solution. The kind of chaining with which the Skinnerians have dealt (i.e., adding new S-R units to an already functioning chain) does not exhaust all the problems associated with the horizontal processes of problem solving. Of particular importance to problem solving is the *spontaneous* integration of separate habits which occurs when an organism infers the consequences of combining previously independent S-R units. This kind of chaining was investigated in Kohler's (1925) classical studies of insight and in the more controlled reasoning experiments of Maier (1930). More recently the authors (Kendler & Kendler, 1956, 1961; Kendler, Kendler, Pliskoff, & D'Amato, 1958) have tried to identify some of the important variables that enable children to combine separate experiences in order to solve an inference-type problem. Much of the research reported in this paper will be concerned with how mediated stimulus and response events aid in the formation of problem solving chains.

The assumption of vertical processes, i.e., the organism responds several different ways at any one time, is also not a novel one. Every psychologist is aware that organisms make several different responses simultaneously, although typically only one is attended to. Sometimes the different responses are interrelated, as is the case between the heart and respiration rates of a fearful organism. In other cases the different responses are independent, e.g., a person's conversation is uninfluenced by his tugging at his ear lobe. The best laboratory example of vertical processes, and one that has much relevance to problem solving, is shown in Figure 1. Those familiar with introductory psychology textbooks will recognize this diagram as representing classical conditioning. Notice that the two solid lines indicate independent S-R units which are operating simultaneously. One is the tone that initiates the "investigatory" response, and the other is the food which elicits salivation. Initially these two associations operate in a *parallel* fashion, but as a result of their simultaneous occurrence an *interaction* takes place which is expressed by the broken line representing the acquired conditioned response.

Obviously the brief reference to horizontal and vertical processes in which it is assumed fundamental S-R principles operate (e.g., discrimination, generalization, etc.) presents at best the barest skeleton of a model of problem solving. It needs the flesh and skin of experimental facts to give it solidity and theoretical principles to clothe it in scientific respectability. Let us now review some of the progress that has been made in this direction.

CONCEPT LEARNING AND UTILIZATION

Although the primitive model just described fails to generate any research by itself, it does suggest that individual experiments cannot be directed at

problem solving in its entirety. There are too many aspects to this phenomenon. The researcher, in designing an experiment, must scan the entire problem solving process and then focus upon that segment that promises to yield fruitful results and is also amenable to investigation.

For reasons that will become evident, it was decided to compare reversal and nonreversal shifts in a simple concept learning task. Figure 2 characterizes each kind of shift by showing a *simplified* version of an experimental situation used with children. The stimuli (cups) for their first discrimination differed simultaneously on two dimensions (size and brightness). The subject is rewarded for responses to one dimension (e.g., large cup is positive, small cup is negative). The other dimension is irrelevant. After learning the first discrimination, the subject is forced to shift to another response. In a reversal shift the subject is required to respond to the same dimension on which he was originally trained, but his overt choice has to be reversed, e.g., he has to shift from a *large* cup to a *small* one. For a nonreversal shift the previously irrelevant dimension becomes relevant, e.g., black becomes positive after large had been positive.

Buss (1953) reported that college students executed a reversal shift more rapidly than a nonreversal shift. He attributed this superiority to the intermittent reinforcements that retard the progress of a nonreversal shift. For example, in Figure 2,[2] when a subject is making a nonreversal shift from large positive to black positive, he is reinforced when choosing the large black cup in preference to the small white cup. This fortuitous reinforcement of the choice of the large cup helps maintain the size discrimination and hence retards the learning of the brightness discrimination. The reversal shift group, on the other hand, receives no reinforcement of the previously correct responses, since they are 100% *non*reinforced.

This analysis is at best incomplete. The work of Kendler and Vineberg (1954) suggested that adult human concept learning cannot be represented adequately by a single-unit S-R theory in which the external stimulus is directly connected to the overt response. Instead, a mediational mechanism (see Figure 3) is required which assumes that the external stimulus evokes an implicit response which produces an implicit cue that is connected to the overt response.

It would be useful to digress for a

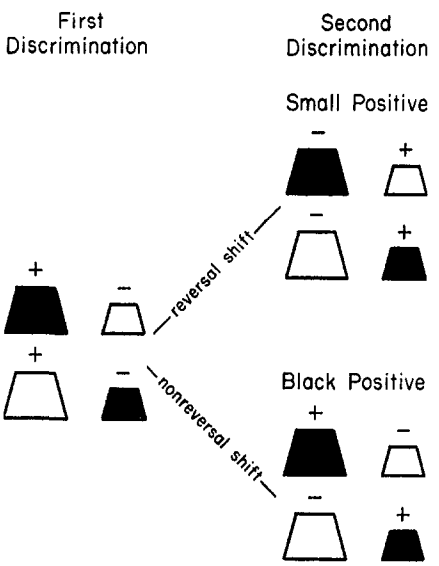

FIG. 2. Examples of a reversal and a nonreversal shift.

[2] The purpose of Figure 2 is to clarify the meaning of both a reversal and nonreversal shift. It would be misleading to believe that it represents *exactly* the methodology of "reversal-nonreversal" studies reported in this paper. For all experiments reported, except that of Buss (1953), designs were used that controlled for fortuitous intermittent reinforcements effects in a nonreversal shift.

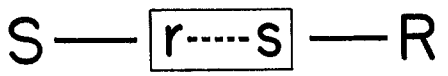

FIG. 3. A schematic representation of the mediational hypothesis.

moment to comment about the epistemological status of these inferred stimulus and response events which are enclosed in the rectangle to emphasize their hypothetical character. Although not directly observable, they are "tied to" environmental and behavioral events. The basic assumption of the mediational hypothesis, at least for the time being, is that the implicit stimulus and response events obey the same principles that operate in observable S-R relationships.

The mediational hypothesis has generated confusion. Perhaps the following brief statements will clarify some possible areas of misunderstanding.

1. The mediational hypothesis is neither new nor revoluntionary. Meyer (1911) and Watson (1913) referred to it, and Hull (1930) gave it a more formal status by coining the concept of the "pure stimulus act." Guthrie (1952) has always laid heavy stress on a mediational-type hypothesis when emphasizing the importance of proprioceptive stimulation in learning.

2. The implicit stimulus and response events *need not* be conceived as having an existence independent of their relation to independent and dependent variables. These implicit events are theoretical constructs. Their epistemological status is closer to such concepts as drive and habit than to directly observable stimulus and response events.

Some mediating events can conceivably and probably will be coordinated to introspective reports, language behavior, muscular movements, and other observable events. Coordinations of this sort can be useful in developing mediational theory. But such coordinations are not *essential* to mediational theory. The fact that genes are not

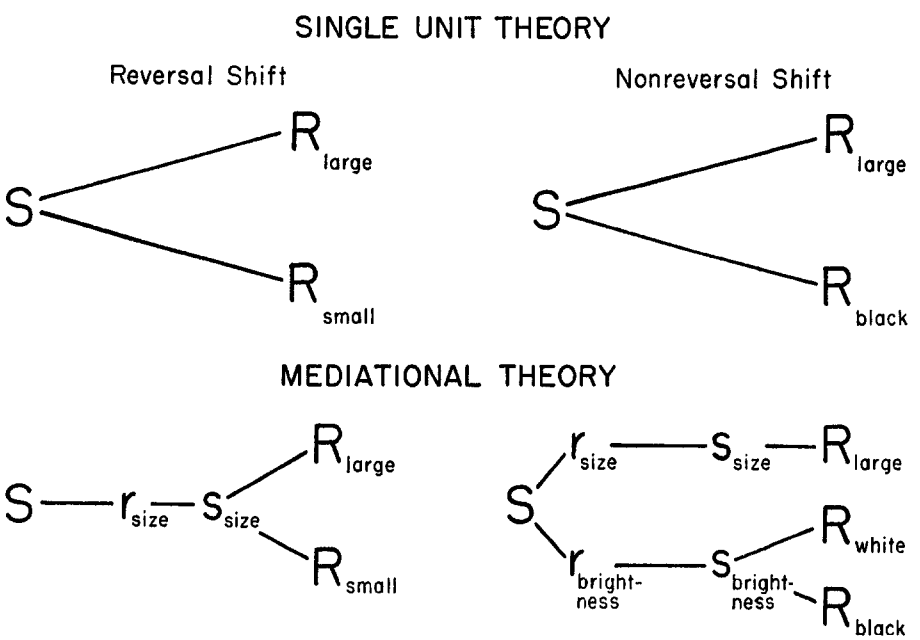

FIG. 4. A single unit and mediational S-R analysis of a reversal and nonreversal shift.

directly observable (at least according to the geneticists consulted) does not interfere with their theoretical and practical usefulness. Even if it were possible to observe a gene directly, it would be necessary to distinguish between it as an observable entity and as a concept within a nomological network. It would be unwise, and strategically shortsighted, to *identify* mediational events with introspective reports or language behavior, or other observable events. The "validity" of the mediational mechanism does not depend on being coordinated with observable events, but depends instead on being utilized in a successful explanatory system.

Figure 4 characterizes reversal and nonreversal shifts in terms of both a single unit S-R analysis and a mediational one.[3] It would be predicted, according to a single unit hypothesis, that if fortuitous intermittent reinforcements were eliminated from a nonreversal shift, it would occur more rapidly than a reversal shift. The reason for this is that at the time of the shift the difference between the strength of the dominant incorrect habit and the to-be-correct habit is much greater for the reversal, as compared to the nonreversal shift. Consequently more training will be required to make the correct habit dominant in a reversal shift. According to the mediational theory the situation is entirely different. A reversal shift enables the subject to utilize the same mediated response. Only the overt response has to be changed. A nonreversal shift, on the other hand, required the acquisition of a *new* mediated response, the cues of which have to be attached to a *new* overt response. Because the old mediational sequence has to be discarded and a new one formed, the nonreversal shift should be executed more slowly than a reversal shift.[4] Thus, if it were possible to eliminate fortuitous intermittent reinforcements, then the stage would be set for a crucial experiment testing the conflicting implications of the single-unit and mediational S-R theories. The results of a series of such crucial experiments (Buss, 1956; Harrow & Friedman, 1958; Kendler & D'Amato, 1955) have been consistent with the mediational formulation in showing that college students execute a reversal shift more rapidly than a nonreversal shift. It is im-

[3] Figure 4 highlights the problem of what are the effective stimuli that are associated to the overt response in both a reversal and nonreversal shift. It is not intended to be a detailed analysis of which there may be several alternatives. For example, in a single unit theory the habit to choose the large container might result from learning two separate specific habits (e.g., the choice of a large black container when coupled with a small white one and the selection of a large white container when paired with a small black one). Another possibility, which would be consistent with Spence's theory (1936), is that the response is to the effective stimulus *large* since responses to the other features of the environment are not consistently reinforced. Similarly adult subjects in a reversal shift might use the mediator *size* or *large* or both. The effective stimulus which is controlling the organism's response must be determined by experimentation. The point made here is that the general implications of the single unit and mediational theories, as discussed in this paper, would be the same for a number of different effective stimuli.

[4] There are two possible ways of analyzing the superiority of a reversal shift over a nonreversal shift within an S-R mediational framework. One is to simply count the number of new associations that have to be formed. As Figure 4 indicates only one new association has to be formed in a reversal shift while two have to be formed for a nonreversal shift. Another possibility is that a mediating response is more difficult to extinguish than is an overt response. For the present the formulation can remain open-ended until information relevant to these two alternatives is gathered.

TABLE 1

MEAN NUMBER OF TRIALS TO CRITERION ON TEST DISCRIMINATION FOR SUBJECTS SCORING ABOVE AND BELOW THE MEDIAN ON THE TRAINING DISCRIMINATION

Group	Performance on training discrimination	
	Above Median (slow learners)	Below Median (fast learners)
Reversal	24.4	6.0
Nonreversal	9.0	15.5

portant to note that in a similar kind of problem rats find a nonreversal shift easier than a reversal shift (Kelleher, 1956). Thus, one is forced to conclude that a single unit S-R theory accurately represents the behavior of rats, while mediational S-R theory is required for the concept learning of articulate humans.

The discontinuity between the behavior of rats and college students directs one's attention toward the conditions responsible for the development of mediational processes. Somewhere on a hypothetical evolutionary dimension between the rat and college student there should be a point where a transition is made from a single unit to mediational control. An obvious place to locate this point would be in the behavior of young children.

A study with kindergarten children (Kendler & Kendler, 1959) showed that these children as a group executed a reversal and nonreversal shift at approximately the same rate. One might conclude that the point in human development was discovered which was psychologically halfway between the white rat and the college student, since the kindergarten children were neither responding in a single unit nor mediational manner, but instead in some compromise fashion. Another possibility is that the children had reached a transitional stage in development, in which the task to which they were subjected led some to function on a single unit basis, and others to operate with a mediational mechanism. If half of the subjects respond in each way, the total results would have revealed no difference between the two kinds of shifts.

The second alternative seems to fit the data. When the kindergarten children were divided into fast and slow learners on the basis of their performance in the first problem (training discrimination), slow learners performed during the second problem (test discrimination) according to the single unit theory; like rats they found a nonreversal shift easier. Fast learners, on the other hand, performed in accordance with the mediational theory; like college students, they found a reversal shift easier. These results were interpreted as demonstrating that these kindergartners, taken as a group, were in the process of developing mediating responses relevant to this task, and that some were further along than others.

If this interpretation be correct, then it would follow that for a group of younger (i.e., preschool) children a still smaller proportion should develop appropriate mediating responses. It

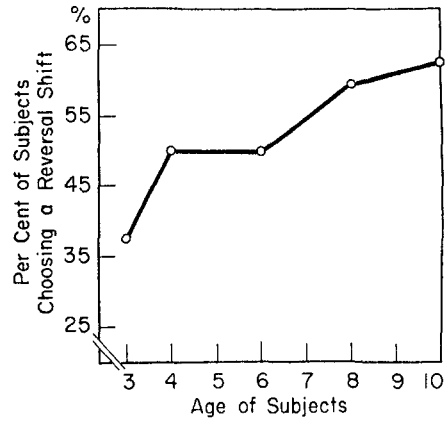

FIG. 5. Percentage of children responding in a reversal shift manner as a function of age.

would be expected that such a group, taken as a whole, would show clearcut evidence of the superiority of a nonreversal over a reversal shift. An experiment (Kendler, Kendler, & Wells, 1960) designed to test this hypothesis produced results consistent with this prediction; like rats, nursery school children found a nonreversal shift to be easier than a reversal shift.

In a very recent study the experimental procedure was modified so that after learning the initial discrimination, the children of 3, 4, 6, 8, and 10 years of age who served as subjects, had a choice of either responding in a reversal or a nonreversal manner. Under such circumstances, it would be expected that the proportion of children who respond in a reversal manner would increase with age. Figure 5 shows that the percentage of children who chose a reversal shift rose gradually from 37.5 at 3 to 62.5 at 10.

Generalizing from all of these results, it would seem that in their early development, children tend to respond in a manner consistent with a single unit S-R theory. With age, they develop a tendency to respond in a mediational manner. The last study cited suggests that it is, or will soon be, possible to ascertain the lawful relationship governing the course of this development.

The point of these experiments is not to classify children into one of two categories: rat-like or human-like. Their aim is to lay the groundwork for experiments designed to investigate the mediational process itself. If one wants to investigate mediational processes, does it not seem sensible to scrutinize them at the time when they are developing? Answering this question in the affirmative, it was decided to investigate the relationship between the hypothesized mediational processes and verbal behavior—a relationship everybody assumes to be intimate and important.

Particularly relevant to this attempt to coordinate verbalization with mediation were observations that during the course of the experiments just described, it was not uncommon for children to verbalize spontaneously the correct solution while simultaneously making an incorrect choice. A few children did this for many consecutive trials. This observation is relevant to the concept of vertical processes. Two chains of habits are occurring simultaneously. One has to do with verbal response; the other with the overt choice. For these children the two chains are parallel, that is, they do not interact.

Luria (1957), the Russian psychologist, made somewhat similar observations in his research with children. He explains this sort of phenomenon in the following way:

> In the early stages of child development, speech is only a means of communication with adults and other children. . . . Subsequently it becomes also a means whereby he organizes his own experience and regulates his own actions. So the child's activity is mediated through words (p. 116).

These observations and their interpretations of noninteracting parallel processes point to the complex interrelationships existing between verbal behavior on the one hand and problem solving on the other. If nothing else, they destroy the illusion that it is reasonable to describe an organism as verbal or nonverbal without considering the problem with which it is confronted. The terms verbal and nonverbal become meaningful—and fruitful—when related to specific problem solving tasks.

It would seem fruitful to investigate the cue function of words for children of two age levels. One possibility is that age influences problem solving

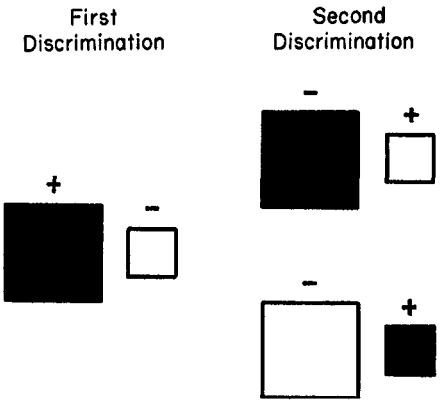

FIG. 6. The experimental procedure used to study the influence of verbal habits on a reversal shift.

only in so far as it leads to the acquisition of words. If younger children, say 4 years of age, could acquire the same words as 7-year-olds, they would solve a simple concept-learning problem the same way. The other possibility is that the acquisition of the verbal label by itself is not sufficient; the word must be integrated with other behavioral chains to influence problem solving behavior. And for this to happen some developmental changes must first take place.

In order to test these two alternatives, children of 4 and 7 years of age were presented with another variation of the reversal shift problem as shown in Figure 6. They initially learned a simple discrimination between a pair of stimuli that varied simultaneously in size and brightness. In the illustration provided in Figure 6, the large black square is correct. While they were learning, the children were required to verbalize aloud the stimuli to which they were responding. One-third learned to say "large" (or "small" as the case may be) by the simple device of instructing them to tell the experimenter which was correct, the large or the small one. Another third learned to say "black" (or "white") in a corresponding way. The remaining third was not required to say anything. After learning the discrimination, all subjects were presented with a reversal shift. In the example depicted in Figure 6, the shift is to small regardless of size. Thus, the group that initially described the correct stimulus as "large" had verbalized the relevant dimension. The verbal response of "black" was irrelevant to this reversal shift.

Figure 7 shows the results of the three experimental groups for the two age levels. If developmental processes affect the utilization of verbal responses in problem solving, then it would be expected that the three verbalization conditions (which produced a significant main effect) would influence the behavior of the two age groups differently. These results suggest, but not quite at a significant level, that there is an interaction effect. Figure 7 shows that the younger children profited by making the kind of verbal response appropriate to a reversal shift, while they were hindered by learning

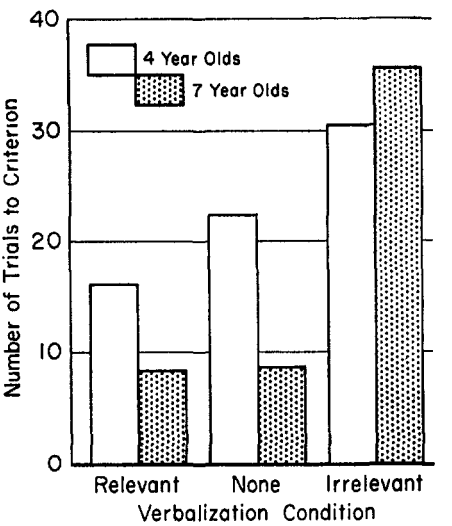

FIG. 7. The effect of verbalizations on a reversal shift for 4- and 7-year-old children.

inappropriate verbal responses. With no verbalization the 7-year-old children who presumably were responding largely in a mediational manner, accomplished a reversal shift much more rapidly than their younger counterparts. But unlike the 4-year-olds, they did not profit from being trained to make the relevant responses. At 7 years of age they are capable of making the response themselves and outside help appears to be of little use. In contrast, the influence of irrelevant verbalizations is marked. The performance of the 7-year-olds was even poorer than that of the 4-year-olds, suggesting that the interfering effects of being given an inappropriate mediated response are greater when one is capable of spontaneously generating the correct one (7-year-olds) than when one is not (4-year-olds).

How are these data to be explained? Attributing differences to developmental factors is not sufficient. It is necessary to represent developmental differences in terms of the concepts of the behavior model that is being used. That is, if a verbal label for a young child does not possess the same cue function as it does for an older child, then it becomes necessary to specify how and why this comes about. To some extent this has been done by emphasizing the transition from a single unit to a mediational system, as well as suggesting that with age an increase occurs in interaction among chains of different vertical levels. But obviously this analysis of the developmental process demands further theoretical and empirical development.

These studies are intimately related to the oft-reported finding that many species of subhuman animals are able to make a fairly rapid reversal shift *if* they receive a previous series of such shifts. Rats (Buytendijk, 1930; Krechevsky, 1932) show a marked improvement in executing successive reversals. They finally reach a point (Dufort, Guttman, & Kimble, 1954), in a T maze, where they learn to go to a new rewarded goal after making only one error. Even more dramatic are the rapid discrimination reversals exhibited by Harlow's (1949) monkeys. But fish (Wodinsky & Bitterman, 1957) exhibit only a slight improvement in successive reversals, while isopods (invertebrates) show no improvement (Thompson, 1957).

Because of the necessity to use somewhat different experimental procedures for different species, it is difficult to draw an unqualified conclusion about the ability of different species to transfer what has been learned from previous reversal shifts to a new one. But the suggestion is strong that as you ascend the evolutionary scale organisms acquire a greater capacity to generate cues that enable them to make rapid reversal shifts. This behavior, according to our analysis, borders on the language responses of humans. The main difference is that our human subjects, except those of a very young age, exhibit rapid reversals without any previous reversal training. Whereas the human automatically seems to generate a mediated response that provides the basis for his rapid reversal, the animal subject must gradually acquire an ability to respond appropriately to some response produced cue resulting from nonreinforcement of a previously correct response.

Up to now, the reversal and nonreversal technique has been used to investigate mediational and developmental variables. It has proved sufficiently flexible to be used in a study (Kendler, Glucksberg, & Keston, 1961) which was designed to lengthen a problem solving chain so that the interaction between various segments could be observed. In this study a

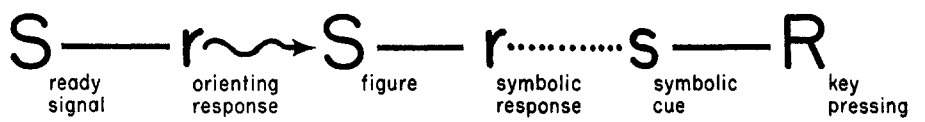

FIG. 8. The hypothesized behavioral chain operating at the time the subject was being shifted to the second concept. (Capital letters refer to directly observable stimulus and response events, while small letters refer to those that are inferred.)

perceptual orienting S-R unit was added on to the mediational chain already described. Figure 8 illustrates in an oversimplified manner the behavioral sequence involved in this study in which subjects had to learn to press the correct button when two physically discrete and spatially separate stimulus patterns were projected on a screen at such a rapid rate that only one could be perceived on any trial. During the learning of each of two successive concepts (involving either a reversal or nonreversal shift), the subject had to pay attention to the relevant stimulus pattern while ignoring the irrelevant one. Thus, in order to make the correct overt response consistently, a subject initially had to make the appropriate orienting response in order to perceive the relevant stimulus pattern to which he had to make the correct mediational response which served as the cue for the key-pressing act.

An experimental design was used in which, at the time of the shift from the first to the second concept, one group had already learned the appropriate orienting response as well as the appropriate mediating act. They needed only to learn a new terminal key-pressing response. The shift, for them, was easy to make. In contrast, the behavior of three other experimental groups was significantly worse. One group had to learn a new orienting response, e.g., look to the left instead of the right. Another group had to learn a new mediated response (i.e., they were required to make a nonreversal shift). The last group had to acquire both a new orienting and mediated response. The fact that the groups which were missing one or both of the necessary behavior units (orienting and mediated responses) did not differ significantly among themselves, as well as being much poorer than the group that had both, highlights the problem of synchronizing the S-R units in a behavioral chain. The advantage in this study of having one appropriate unit without the other is at best negligible. The reason for this is that reinforcement is only achieved consistently when both the appropriate orienting and mediating responses are operating. This particular study points to the need for discovering laws associated with the strengthening and weakening of independent S-R units in a problem solving chain, as well as the principles governing their synchronization.

This study also highlighted a very basic problem in all of these reversal studies. This problem has to do with the very first correct response following the reversal shift. After discovering that the previous mode of responding is erroneous, what makes the subject change his response, i.e., push the button that was previously wrong? Introspective reports fail to provide any clearcut answer and even if they did they would be in need of explaining (Kendler, 1961).

One hypothesis is that the selection of the new correct response is due to the operation of a behavioral chain in addition to the one described in Figure

8. The first nonreinforcement in a reversal shift sets off a chain, the consequence of which is to select the response other than the one that was previously correct. This may result from a number of different reasons (e.g., logical considerations, forgetting, etc.). The important point, however, is that the new key-sorting response occurs contiguously with the implicit mediational response appropriate to a reversal shift. As a result, a new association is formed between the old implicit cue and the new key-pressing response.

In essence, what is being stated is that adult subjects, when making or deciding to make the first correct postshift response, do not adopt the *principle* underlying a reversal shift. Instead, it is assumed processes are operating which encourage the selection of the correct response while an implicit cue appropriate to a reversal shift is operating. This sort of an analysis was described previously (Kendler & Mayzner, 1956) as

> sort of a James-Lange theory of problem solving . . . one makes the overt correct . . . response and if the appropriate symbolic cue is present, then problem solution will occur (pp. 247–248).

Guthrie (1959) says the same thing more neatly: *"What is being noticed becomes a signal for what is being done"* (p. 186).

Again the authors would like to guard against giving the impression of oversimplifying a terribly complex problem. They do not believe the contiguous occurrence of an implicit cue from one chain with the correct overt response from another tells the whole story. This new association in order to persist must be reinforced and in some manner "fit into" the subject's ongoing behavioral chains.

The emphasis on this vertical connection between a cue and a response from different chains is related in a distant way to Hebb's (1958) stressing the role of "chance" in problem solving:

> There are few scientists who have not had the experience of setting out to solve problem A and ending up instead with the solution to B. . . . This is serendipity, the art of finding one thing while looking for another (p. 215).

According to the present analysis, serendipity results from the adventitious and contiguous occurrence of a cue and a response which are themselves segments from different behavior chains. Theoretically it should be possible to demonstrate this point experimentally by training subjects to respond simultaneously to two separate tasks. A problem then would have to be presented that requires for its solution the combination of a stimulus from one chain with the response from the other. In such an experimental situation, controlling the time relationship between the two should have an important effect on problem solving. Presumably contiguity between the two should provide the most optimal conditions for problem solving (Underwood, 1952). The development of this kind of experimental procedure should allow for parametric studies of the basic variables of the phenomenon which has commonly been called "insight," as well as throw light upon issues raised by others (e.g., Cofer, 1957; Maltzman, 1955; Saugstad, 1957).

The pretheoretical model that guides the present research has many more facets that can be exploited. Only one will now be mentioned. Glucksberg (1962), for example, extended neobehavioristic drive theory (Spence, 1956) to problem solving. He used a functional-fixedness problem (Adamson, 1952, Duncker, 1945) in which the correct response in the habit hier-

archy could either be made to be low or high. If the correct habit was low, it would be expected that a strong drive would retard problem solving because it would retard the extinction of the dominant incorrect response (Kendler & Lachman, 1958; Perin, 1942). Since drive energizes behavior, a high drive should facilitate problem solving performance when the correct habit is dominant. The findings were consistent with this analysis.

Because functional-fixedness problems are often represented in perceptual terms, Glucksberg was interested in seeing whether the same drive model could be applied to a simple perceptual recognition problem in which subjects were instructed to identify tachistoscopically presented words as rapidly as possible. The results were similar to those reported for the functional-fixedness study: when the correct response was dominant, an increase in drive improved performance, i.e., the visual duration threshold was lowered. In contrast, increasing drive when the correct response was low in the hierarchy raised the threshold.

There is obviously still much more work, both empirical and theoretical, needed to develop the model that has been described. At this point it may be appropriate to summarize the major points of this paper.

There is not just one way to investigate problem solving. The researcher who is interested in problem solving has several different pretheoretical models from which to choose. This paper reported the results of a research program based on an S-R model in which the importance of horizontal and vertical processes were emphasized. Horizontal processes refer to the linking of successive S-R units into a behavioral chain, while vertical processes refer to the assumption that independent chains occur simultaneously. A series of experiments was reported, the implications of which supported postulating a mediational mechanism within a behavioral chain. By comparing the behavior of human subjects of different ages, as well as relating their results to lower animals, it was possible to infer that as a child matures he makes a transition from responding on the basis of a single unit S-R mechanism to a mediational one. Additional data were cited that suggest the full impact of verbal behavior on problem solving depends on developmental processes that encourage interaction between chains at different vertical levels. It was also suggested that problem solving begins in a simple concept learning task when a correct overt response from one behavioral chain occurs contiguously and adventitiously with the appropriate implicit cue from another chain. The paper was concluded by citing findings that suggested the neobehavioristic drive theory which assumes that the effect of different levels of drive depends on the position of the correct response in the habit hierarchy is applicable to a functional fixedness problem as well as a perceptual-recognition task.

If nothing else, it is hoped that the present paper demonstrates that it is possible to investigate problem solving in a systematic fashion. If more psychologists accepted this possibility and were willing to expend their research energies in the field of problem solving, progress in this area would be greater than it is today.

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