tention are customarily explicitly described in a mathematical way. In the same way, all arguments are potentially mathematical in character, but some arguments use mathematical terminology and categories in an explicit way. Thus we have two questions: that of significant uses of mathematical concepts, and that of mathematical argumentation.

(a) In what common situations is mathematics used? If mathematics is defined as the deductive investigation, with predictive consequences, of quantitative models representing human fields of attention, we can isolate certain modes of behavior within a culture which are clearly mathematical. For example, the reckoning of money in order to pay taxes is clearly a mathematical activity. It uses the quantitative model, namely, amounts of money, which represents economic value, or purchasing power. It deals with available money in a deductive way, with the intention of showing how the taxes can be paid. This is mathematical behavior, where, for example, eating rice is not, even though the rice within the latter field of attention can be and often is measured and valued.

A broad study of the culture to determine those areas wherein mathematics is put to use is thus important. In this way not only can examples be found which would be useful for teaching, but also generalizations can be made about the overall ways in which mathematics functions within the group. In this connection, the content-centered dictionary of culturally relevant materials should be subdivided into those materials which are commonly subject to mathematical manipulation, and those materials which are not put to mathematical use.

From this study, certain mathematical operations will be discovered to recur in various different circumstances. For example, it is clear that the operation of addition of small whole numbers occurs with such frequency in all societies

that it must be considered universal. The numerical upper bounds of addition problems considered possible differ from culture to culture, however, depending on the types of materials which must be added in everyday use. For another example, it is clear that operations with fractions are not commonly found in technologically undeveloped societies. It should be possible to suggest generalizations as to where such operations with fractions appear, particularly giving the material circumstances which seem to make these operations necessary.

In a given culture, therefore, everyday uses of mathematics should be tabulated and those uses with common features should be classified together. In this way, an organized presentation of mathematical behavior can be prepared for the given culture, and used to suggest ways of improving mathematics teaching, as well as to broaden the understanding of the culture involved. And from the study of many such societies, generalizations can be made as to what operations and activities are universal, and which are limited by special circumstances. These generalizations should then be tested within cultures where the special circumstances are absent as well as others where they are present.

(b) What mathematical argumentation is used? This question is concerned particularly with the reasoning used in the mathematical behavior observed in the culture. It has already been recommended that forms of argument of all types be studied and classified. The results of that study should then be used to help analyze the special case of mathematical argumentation. In order to be considered, arguments must appear in the linguistic behavior which accompanies mathematical activity, and they must have cultural relevance and value.

It may be, of course, that in a given culture the arguments which support mathematical conclusions are not the arguments which would be used in a western society.

It is highly unlikely, for instance, that the formal proof structure which was introduced by Aristotle and Euclid, among others, will appear in undeveloped cultures. But there will be arguments, some of which can be put in standard form, which are commonly used in mathematical situations. These must be listed, classified and analyzed.

(3) Psychological

We are concerned not only with the place of mathematics in the language and in the culture, considered broadly, but also with the reaction of the individual to mathematical features of the various fields of attention upon which he focusses. It is possible that the individual may refer to a specific everyday situation in language showing the fact that he performs, or does not perform, a certain mathematical abstraction. For instance, two persons may respond to a presentation of three rows each with four chairs, in different ways. One may see them as twelve chairs, without observing the organization of chairs. The other may see them as three rows of four chairs, and infer that there are twelve chairs altogether. The second person has made a mathematical observation not noticed by the first. This may happen for a number of reasons, all of which should be investigated. Thus there are two questions which must be asked concerning the reactions of individuals to their environment. The first concerns the mathematical distinctions and abstractions made by these individuals, and the second concerns the reasons underlying their reactions.

(a) What individual reactions are mathematical in character? After linguistic behavior and social usage have been catalogued and classified, it is necessary to investigate the response of individuals to certain situations. The situation must be set up to investigate those areas of behavior and language which have mathematica

implications, but which are not adequately explained by a gross survey of the linguistic and anthropological features described above. For instance, simple multiplication involving small whole numbers is apparently a common feature of all societies. However, it is not clear when this operation is actually performed, and under what circumstances. In the example of the chairs given above, the first person does not see it in terms of multiplication, but instead counts the chairs as if they were randomly organized. Such behavior can be studied, by setting up carefully controlled psychological experiments, and the responses of an appropriate sample of the population tabulated and compared with reactions of a similar sample of another culture.

Moreover, those examples of mathematical behavior which are known to be present in the language and in the material culture of two societies can be compared in certain quantitative ways. For instance, individuals in two cultures may sufficiently attend to geometrical shapes to be able to assemble a simple jigsaw puzzle, but those in one culture may on the average assemble that puzzle more rapidly than members of the other culture. Such quantitative tests are very useful, since they indicate not only the presence or absence of a particular concept or mode of behavior, but also indicate the degree or strength of its presence. Examples must be chosen which will measure the comparative behavior of individuals in two cultures in as many mathematically relevant areas as possible. These examples will, of course, be selected on the basis both of the disciplines which it is desireable to teach and on the observed linguistic and active behavior of the individuals who are to learn.

As a result of this inquiry, it will be possible to make suggestions for more effective teaching. In particular, those concepts and activities which are most

strongly present in the culture can be used as starting points for building mathematical bridges to the less well recognized and less used concepts and activities. Moreover, the ways in which these concepts and activities are recognized, verbalized and used should suggest means to teach other concepts and activities.

(b) What mental and emotional factors underlie these reactions? This is, of course, a most difficult question to answer, since it involves digging deep into the role of the individual within his culture. It involves careful observation of his values, fears, motives, desires and experience, both at the present and during earlier years. We know very little about these deep emotional factors in the psyche of the individual, even within our western culture, which has been analyzed so thoroughly. Thus the problem is doubly difficult within the less developed non-western societies. But the problem can and must be approached, if we are to make adequate recommendations for better mathematics teaching and learning.

For instance, the reaction of school age children to mathematics instruction depends very strongly on their emotional development. It may be that a child was beaten in an early grade for not knowing a certain mathematical fact, and, as a result, bears a hatred for the subject which hinders his subsequent learning. There may be factors of this type which affect not only individuals but also social groups, and these factors must be discovered. Children who are told not to question the authority of their elders will tend not to have that inquisitive type of mind which is so necessary to good performance in mathematics.

The values which are built into the culture are equally important to this study. A society which places a low value on originality and analytic thought is not likely to produce many mathematicians. And where the prime value is placed on obedient reception of that which the tradition bequeaths to the new generation,

creative mathematics is even more likely to suffer. That which is most desired also influences the child's response to education. If the trader holds a position of social pre-eminence, it is likely that business skills will be sought after by the growing child. The values, desires and motives of a culture must be explored thoroughly, particularly as they are related to mathematical concepts and activities. If we know and understand them properly, we can use them, not only to gain a deeper understanding of the community with which we are working, but to prepare curricula and texts which are suitable to the children of that community.

(4) Educational

All these aspects of the problem come to focus in the educational problem, which itself can be divided into three subproblems: how are mathematical concepts learned by individuals within the culture; how does the culture itself learn new behavior patterns; and how open is the culture to outside influence. These questions, as was also the case with the other questions considered, are as applicable to a tribal society as to a transition society, and it is necessary to consider both societies in relation to the incoming western culture.

(a) How do individuals learn mathematical concepts? It is clear that every society has certain mathematical ideas which are part of the language, the material culture, and the individual psyche. We must explore these ideas, in accordance with the suggestions made in the previous sections. But we must also explore the procedures which are followed within the culture to teach children these ideas. It may be that they learn them simply by observation and imitation. It may be that there are definite teaching techniques used by the older people of the group. We know very little about this matter in western society outside of the classroom, and even less about the matter in non-western societies.

It is possible in approaching this problem to use both the memory of older people in the town, when they look back on their childhood, and the actual experience of the younger children. It will require observation on a fairly extensive scale, however, to do much useful work with young children, since much teaching is incidental and not planned. But, if possible, experiments should be set up, using those concepts which the previous studies of the linguistic, anthropological and psychological phases of pre-mathematical behavior have shown to be present, in order to obtain useful information on the ways in which these concepts and procedures were learned. Hopefully, these experiments will allow cross-cultural comparison and contrast.

(b) How does the society learn new behavior patterns? The assumption here is that no society can be entirely static, even though some societies change much less rapidly than others. There are always new situations arising, which require an intelligent response if the society is to continue to function with the same vigor as previously. For instance, there may be an unusually dry season, or the harvest upon which the food supply depends may be poor. It is important to observe the reaction of the group to this difficulty. Wherein are living patterns changed to meet the crisis? The new response, which must be learned lest serious trouble result, comes from the application of intelligence to the situation. Of course, in some cases there may be a traditional response to the difficulty. Care should be taken in the study to isolate such traditional responses and see what other responses in related situations display originality and intelligence.

Certain of these new responses may be mathematical in character. Where the society is required to change its behavior patterns in ways that involve mathematics, special care should be taken to analyze the procedures used in making

that change. The investigator may not, of course, be fortunate enough to identify such situations, in which case he will have to depend on learning patterns derived from less critical cases, and apply the result to mathematics learning.

(c) What is the response of the society to outside influence? This is a special case of the question asked in the previous section. Particularly at the present time, rapid social mobility and change require old cultures to make responses to new and unexpected incursions by alien cultures. This is, of course, the basic problem of this whole study, since western culture has come into non-western societies with a vast program of education and social reorganization. We are here concerned primarily with the acquisition of mathematical knowledge and skill, but this same problem parallels all other problems raised by the aggressive western culture.

It is thus important to observe closely how the group under consideration responds to this or any other invasion by outside forces. Where a new food crop is introduced, how effectively does the group make use of it? Where the economic structure changes, how adequate are individuals in the face of different trading and employment patterns? Where technological advances threaten traditional customs and tools, are the traditional patterns replaced by new ones which are better, do new patterns regardless of their value overwhelm the old, or do the traditions hold on tenaciously despite their evident inadequacy?

The answers to such questions as these will illuminate the problem of teaching a new subject matter or a new approach to an old subject matter. Culture clash and culture change is the theme of this research project, which is thus aided to a solution by any examples of this change. An expanding, flexible, aggressive social group is more likely to make effective use of the new knowledge

than is a retreating, inflexible, defeated group. Yet there are avenues of approach even to the defeated group, or at least to individuals within it who can be detached from their larger culture. Change is inevitable in the present world and one of our tasks is to make that change as meaningful, vital and hopeful as possible for those people caught up in it.

c. Conclusion

Within every society mathematical and logical thought functions to at least a limited extent. This study suggests that the educator who approaches any given society find those aspects of his discipline, in our case mathematics, which he feels are most basic. We have here suggested certain areas within arithmetic, geometry, logic and applied mathematics as of basic significance to our society. The educator must then state for himself his goals in clear--and at the same time minimal--terms, so that he can effectively attempt to achieve these goals in teaching individuals within the society to which he comes.

We have also suggested that the educator acquaint himself thoroughly with those aspects of the society to which he comes which are relevant to his subject matter. In any given discipline, every society has some background which can be analyzed within the categories of linguistics, anthropology, psychology and education. In the case of mathematics, we require information on pre-mathematical and pre-logical behavior in each of these areas. And then, with this information the educator can proceed to devise better texts, curricula and teaching techniques better in the sense that they communicate more easily and effectively to the learner within that culture. These texts, curricula and techniques must build bridges from the information and procedures known within the culture to the information and procedures deemed desirable for it to learn. If these bridges are

built, it is hoped that much of the mechanical and meaningless teaching now observed will be eliminated, and that the learner will receive an education which is useful both within his own culture and within the new culture into which he is entering. Moreover, if his education is truly adequate to his needs, he should acquire tools which will enable him to deal creatively with new problems which arise after his formal education is complete. The final, desired end-product of education is thus an individual who is able to push the limits of knowledge beyond the place they were when he began his education. Because of his achievements, his society is then a part of the world-wide open society, no longer only a passive receiving culture. Our problem is solved at that point, because it no longer exists.