CHAPTER 2—STATEMENT OF THE PROBLEM

In the previous chapter, I attempted to state the underlying reasons for the present study. These reasons are, however, very general in character, and it is thus necessary to bring them into much sharper focus. The overall problem is clear, namely, how to improve mathematics teaching in cross-cultural situations. But this problem, thus stated, is too vague to allow fruitful investigation.

Thus in this chapter, I will state, as precisely as possible, two general problems, and then suggest subdivisions of these problems which are suitable for detailed study. The first general problem is the preliminary question of determining the basic areas of mathematical and logical thought to be studied. The second general problem is that of obtaining specific information about a culture in order to understand the role of pre-mathematical and pre-logical thought in that culture.

a. Basic Areas of Mathematical and Logical Thought

This problem is of great importance to the present study, since it will show us a direction in which to go. Thus it is fortunate that much of the work in this area has already been done by those concerned to set up rational syllabi for elementary mathematics instruction. This study thus can draw upon the fruits of these previous investigations, in order to have basic information upon which to base its study of pre-mathematical and pre-logical thought. In this enterprise, the traditional division of mathematics into arithmetic, geometry, logic and applied mathematics has been followed.

(1) Definition of mathematics

Before going into each of these specific areas, however, it is important that we have a working definition of mathematics itself. To this end, we state that mathematics is the deductive investigation, with predictive consequences, of quantitative models representing human fields of attention. This definition deserves a brief explanation at this point, as well as detailed exposition in the course of the study.

In the first place, mathematics is concerned with human fields of attention, at least in its beginning stages. It is true that mathematics soon constructs its abstract edifice and leaves human experience far behind, but the edifice is always constructed from materials which have analogues in human experience. The term 'field of attention' refers to the fact that an experience has a form as well as a content. Mathematics, of course, is more concerned with the form than it is with the content.

In the second place, mathematics makes use of quantitative models which represent these fields of attention. Detailed investigation of fields of attention will show that a certain value-scale is always implied by the manner in which we view a given field of attention. Such a value-scale is subject to numeration, since it is linear and ordered, and this numeration is a quantitative model, which thus represents the field of attention.

In the third place, mathematics is a deductive investigation. In mathematics, the quantitative model representing the field of attention is subject to manipulation, using certain syntactical procedures present in the language and culture. These procedures in western mathematics normally reduce to deductive logic, when they are carried out in full detail, although most often in practice many of the steps in a full proof are omitted.

Finally, the deductive investigation of the quantitative models representing human fields of attention has predictive consequences. The consequences which are drawn, using these standardized procedures, from the models have themselves reference to fields of attention. When, therefore, these consequences are applied to fields of attention, they are said to be valid consequences when they fit the fields of attention properly, and otherwise invalid. This is the predictive aspect of mathematics, since in this way we can make additional statements about the stuff of reality. Strictly speaking, of course, the application of the consequences of mathematical derivation to elements of experience from which the original statements were drawn is applied mathematics. Pure mathematics is content to draw the deductive consequences without testing their applicability. However, we must be concerned in this study with both areas: pure mathematics and applied mathematics.

(2) Arithmetic

It is not necessary here to specify in detail those aspects of elementary arithmetic which are to be explored. Too detailed an exposition of that subject would be only an outline of the curriculum of mathematics in the primary school. Broadly speaking, we must explore operations on sets, whole numbers, place values, inequality and order, operations on whole numbers, fractions, operations on fractions, negative numbers and operations on negative numbers.

It is important to determine the presence or absence of aspects in each of these subjects within the culture being studied. And, where a concept is present, it is important to know the degree to which it is present. It is to be expected, of course, that many of these concepts will be present in limited ways or under special forms.

(3) Geometry

In the same way as in the previous case, we can give only the general outline of geometric concepts whidh interest us. In the first place, we are concerned with recognition of and operation with simple geometrical shapes, such as the triangle, circle, rectangle, straight line, and so forth. In the second place, we are interested in the application of numerical value scales to these shapes, whether in measurement or in counting. Third, we are interested in the use of geometrical figures as models for other mathematical and physical situations.

As before, we must do a detailed study to evaluate the presence and use of these various concepts in the culture. In particular, we must use experiments which enable us to compare various cultures with respect to the use of these concepts.

(4) Logic

In the field of logic, we are interested primarily in two things: how propositions are constructed from simple terms, and how propositions are joined together to form arguments. This information can be obtained through a study of mathematical operations or, more generally, through observation of argumentation and inference in every-day situations.

In each case, we are concerned particularly with the logical connectives which are used. The connectives which are important to present-day logical theory are the following: 'and,' 'or,' 'implies,' 'equivalent,' 'not.' In certain cases these connectives are used to state propositions which are constructed out of simple terms. In other cases they are used to connect propositions into coherent arguments.

(5) Applied mathematics

In this subject we are interested in the ways in which predictive conclusions can be drawn from mathematical statements. For instance, we wish to know how mathematical reasoning is used in trading and marketing, in house-building, in games, and in household budgeting. In general, we must find those areas where mathematics is applied and is useful to the people.

b. Areas of pre-mathematical and pre-logical thought

The rough outline given above makes possible a detailed statement of the places within the culture of the learner where we can expect to find behavior relevant to his learning of mathematics. We see above four general disciplines—arithmetic, geometry, logic, and applied mathematics--which he will encounter in the early stages of his contact with western mathematics. He is prepared for this study, or hindered from it, because of elements within his own culture.

In Chapter 1, we stated that there are four general problem areas wherein difficulties are likely to arise. These are linguistics, anthropology, psychology and education. In this section, we will attempt an analysis of pre-mathematical and pre-logical thought within each of these four areas. In every case, we will call attention to specific questions which research should attempt to answer. These questions should, of course, be answered both for the culture out of which mathematics and logic, as formal disciplines, have grown, and for the cultures of those who are learning mathematics.

(1) Linguistic

There are three basic problems of a linguistic character which must be considered: how are fields of attention described, how are propositions constructed, and how are propositions connected into coherent arguments. For each of these problems, we can make certain general statements which will help in the analysis of the problem within a particular culture group. These statements are, of course, only tentative since they may be proved irrelevant by the study as it progresses.

A hypothetical structure must be assumed in order for inquiry to begin, but the structure must be subject to continuous modification as new evidence is presented.

(a) How are fields of attention described? The first question concerns the description of fields of attention. In this discussion, we understand by a field of attention that to which the observer is paying attention, including anything that is, from his point of view, under observation, and excluding whatever he does not notice. It may be, of course, that another observer who is noting the same situation may observe a different field of attention, because of his different perspective or different preparation. For example, if a city person and an experienced hunter walk together on the same trail, the city dweller will simply not notice those signs of animals which the experienced hunter sees, even though they are theoretically within his range. Thus the field of attention is described from the point of view of the observer himself, and his word with respect to it is final.

A preliminary analysis of the structure of fields of attention indicates that there are five elements present, even though not always verbalized, in every observation. They are the following: the content or stuff out of which the field of attention is made; the form or presentation of the content, that is, the way in which the stuff is displayed; the aspect of the presentation which is considered; the measure which is imposed on the aspect; and the value of the measure. Tentatively, it seems probable that every field of attention has present in it each of these elements. Moreover, it is possible that the human mind imposes these categories by the very process of observing.

Several examples may serve to explain these five structural elements of a field of attention. For instance, if the field of attention consists of people, these people may form a group. Moreover, we may consider the group of people according to number, and we may count by units. If we do so, we may report that there are seven people present. In this case, the content is people, the presentation is a group, the aspect is number, the measure is unit, and the value is seven. For another instance, if the field of attention consists of water, the water may be in a bottle. We may be concerned with the volume of the water, and we may measure by pints, reporting that there are two pints in the bottle. Here the content is water, the presentation is a bottle, the aspect is volume, the measure is pint, and the number is two. For another example, the content may be running, the presentation a race, the aspect speed, the measure miles per hour, and the value twenty.

The problem is to classify the ways the culture has of reporting the fields of attention which are experienced by individuals within the culture. The names given to the content will make up an experience-centered dictionary of culturally useful and meaningful materials. More significant for us, however, are the names given to presentation, aspect, measure and value. We will find certain general terms in each of these categories, and we will find that we can organize and classify the content names within these categories. For instance, we find in English a fundamental split between countable and non-countable names, and we find that this split affects all five categories: content, presentation, aspect, measure and value.

One classification which will result from this study is the classification into more and less general terms. For instance, the word 'tree' is more general than the word 'mahogany', and the word 'set' is more general than the word 'congregation'. The classification of terms will parallel the taxonomic distinctions of, for example, botany, into genera, species and other categories. Such a classification is inherent in the language, and can be determined by purely linguistic techniques, and is clearly pre-mathematical and pre-logical in character.

In our classification of terms descriptive of fields of attention, we must consider fields of attention at first without any restriction on the content, and determine the most general terms for reporting the content and structure of any field of attention. We must then successively restrict the content, as well as structure, of the field of attention, in order to find successively more limited terminologies. We must then attempt to state the role of these various terminologies in pre-mathematical and pre-logical behavior. Thus, for instance, we find in English that the term 'set', which is on the second level of generality as a presentation-word, since it refers only to countable fields of attention, is a term of great importance in the development of mathematics.

(b) <u>How are propositions constructed?</u> The second basic linguistic question concerning pre-mathematical and pre-logical behavior takes up the matter of how terms, which themselves are descriptive of fields of attention, are organized into propositions. A proposition is a statement which either affirms or denies, and is limited to one sentence. We will consider in the next section how propositions are organized into arguments, and thus the limitation to one sentence for a proposition is useful and meaningful.

A few preliminary remarks can be made about the nature of propositions, remarks which may be applicable to all languages. Of course, these general remarks may be proved wrong by examples drawn from languages outside the present study, in which case modifications are necessary in the generalizations underlying these remarks. The first such remark is that propositions are constructed out of more elementary terms, some of which are descriptive of fields of attention, while others have an organizational function, and still others combine both organizational and descriptive functions. Thus all the terms which are descriptive of fields of attention can enter into propositions, where some may serve a purely descriptive function, while others also perform an organizational function. We must therefore state of such terms what roles they play in propositions. We must also locate and state those terms in propositions which are purely organizational in character.

The second remark is that propositions can be atomic or molecular. An atomic proposition asserts something to be true of the content of a field of attention. The content of the field of attention is called the subject of the atomic proposition, and depends, as before, strictly on the point of view of the perceiver. That which is affirmed of the content of the field of attention is called the predicate, which also depends on the point of view of the perceiver. The predicate of the proposition may assert something true within the given field of attention, or may relate that field of attention to another field of attention.

The predicate of a simple proposition may be simple or complex, depending on whether one or more attributes are asserted of the subject. The portions of a complex predicate can be connected in a number of different ways, and it is important to list and classify these connectives for a given language. Some of them coordinate attributes of the same class, while others correlate attributes of different classes.

The molecular proposition is a proposition which has at least one complete atomic proposition as a unified, proper, sub-portion of itself. This sub-proposition is thus modified as a whole within the molecular proposition. It may be the only sub-proposition within the molecular proposition, as in the case where an atomic proposition is negated. Or it may be one of two or more atomic sub-propositions, as in the cases when the logical connectives 'and', 'or', 'implies' or 'equivalent' are used. We must study what logical connectives are used in a given language, and classify them in a way which is proper to that language.

The third remark is that propositions are customarily stated to be true or false. In order to determine the truth-value of logical connectives, we must note the truth-values of selected atomic propositions, as well as the truth-values of molecular propositions which can be formed from them, and from this information find any regularities which may appear. If the connective displays a different pattern of truth-values depending on the contents of the atomic propositions, then we say that the connective does not have truth-functional value. The connectives which are most significant for pre-mathematical and pre-logical thought are, of course, those which have truth-functional value, since they behave in a consistent and formally predictive fashion.

From these remarks concerning the construction of propositions, and their verification and amplification for a given language, much can be learned which will suggest ways to organize the learning process. It is not possible, for example, for a student to learn if he is forced to use propositional forms which do not appear in his language, or which do not have the same truth-functional value in his language. Moreover, if new patterns must be taught, which are not present in the learner's language, then the bridges which cross this gap must be built, using proposition-forms which he does know as the materials for constructing the bridges. It is highly unlikely that a proposition in English cannot be converted, by suitable transformations, into a proposition using familiar forms in another language.

(c) How are arguments constructed? This third question asks how propositions are organized into coherent arguments. The answer to this question depends upon the opinions of members of the society as to what constitutes coherent arguments. It is a common experience that arguments which seem persuasive to members of one community seem disconnected and fallacious to members of another community. Even within the same group, one person may find an argument compelling which another person rejects entirely as unconvincing. The second man might reject it because he does not understand it, or because he finds flaws in its construction--but it is the fact that he rejects it that is important in teaching. The teacher is of course persuaded that his own arguments are correct, and wishes his student to learn the correct form of argument. But, even where he knows that he is correct and the student incorrect, he must use at first the forms of argument which his student uses.

An argument can be defined for the present as a connected, linear series of propositions which lead from a beginning to one or more conclusions. Normally, in such an argument, a conclusion, whether it be intermediate or final in the argument, depends in some way on the statements which precede it. We say that the earlier statements imply the later statement. Thus we can symbolize the argument as a series of single propositions of the form 'p and q and r and ... imply z,' where p, q, r, ... , and z are all propositions.

It is normally difficult to state the reason for the implication in a proposition. The point at which one person may draw a conclusion may be the same point where another person requires a further argument. The first step, therefore, toward analyzing the ways in which a particular society constructs arguments, is to take a series of arguments, and isolate the points of implication. The propositions which precede the implication and that which follows it must be isolated and considered in relation to each other. If many such arguments are analyzed, certain salient features will appear, which will suggest generalizations valid for the particular culture. These features may include the appeal to tradition, the appeal to experience, or the appeal to authority, as well as the appeal to purely formal implication.

On the basis of such a typology of arguments, generalizations can be made concerning the usefulness of certain types of argument in bridging the cultural gap. In every case, it should be made clear both to the student and to the teacher what type of argument is being used, and what type of implication is acceptable. Hopefully, the pupil can be taught to use arguments which are productive of results. But, before this can happen, he must at least learn to recognize the types of argument most common in his own culture, know their limitations, and make use of them when they are needed.

(2) Anthropological

This section of the problem consists primarily in the application of what was stated in the preceding section. As we indicated in Chapter 1, anthropology helps us to understand the meaning of the terms we discover through linguistic analysis. Without the aid of anthropology, linguistics can analyze nothing but sound, to show its formal organization. The discussion of fields of attention is essentially anthropological, since it focusses on that to which the language refers. The statement of culturally meaningful contents, presentations, aspects, measures and values of fields of attention, is in a very real sense, a statement of the culture itself.

But there are questions which we must ask, once the linguistic groundwork is laid in the fashion indicated in the previous section. Every field of attention has its potentially mathematical forms of description, but some fields of at-