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~~Jan. 24-25, 1964~~
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APPENDIX 8.

Quantification and Arithmetic Operations in Kpelle

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This study is occasioned by the observation that the acquisition of mathematical concepts and procedures has generally been more difficult for a Liberian student than for his counterpart in American society or in many other societies. Theoretically, any one or a combination of several factors might be suggested to account for this difficulty. Each possible reason deserves consideration and investigation by whatever methods are appropriate. Some of the more reasonable hypotheses are the following -- with no particular significance in the order in which they are listed:

(1) The fact that elementary mathematical facts and operations are introduced only in English, at a time when English is still a relatively new, imperfectly learned, and difficult language for the student. Comment: If this is a major factor, then closely comparable difficulties might be expected in other elementary subjects such as history, geography, or hygiene. The observation is, however, that mathematics is unique in its difficulty. It may be significant, however, that arithmetic is the first subject introduced apart from those subjects (reading, writing, spelling) which have to do specifically with the English language itself.

(2) The techniques of teaching mathematics may be significantly different from, and inferior to, the techniques used in school systems where less difficulty in learning is found. Comment: While this is almost certainly true, it only transfers the problem to another field; it remains to be explained why Liberian teachers

find it difficult or consider it unimportant to use methods proven effective elsewhere.

(3) The age at which children begin school -- which until recently has been considerably higher than in most countries -- may be such that the beginner has passed an age at which the acquisition of the new "language of mathematics" is maximally easy. Comment: This would be analogous to the fact that children in their teens generally find it more difficult than children aged five or six to learn a second language. This could be tested in part by seekihg a correlation between success in mathematics and age at beginning school. The presence of such a correlation would be significant; its absence would not, since other factors might be inhibiting the learning process for even the most receptive students.

(4) The cultural background from which the students come may be such that they see no reason to be motivated to learn anything beyond the most elementary and readily memorized mathematical facts. Comment: There is, indeed, some evidence that computations involving numbers higher than thirty or forty are uncommon in indigenous Liberian cultures, and that these are taught and memorized very early in life, so that computation is not a strikingly significant part of adult life; however, we are dealing with students who by definition are motivated, as evidenced by their very voluntary presence in school, to learn things not present in their own culture. Furthermore, the concepts and operations with which we are concerned are almost equally strange to the child in other societies when he first meets them in school; though they are present in adult society around him, they are far less conspicuous than such activities as reading, driving a car, or attending church services.

(5) Mathematics may function in the local culture as a specialty practiced by a favored few, who protect their distinctive knowledge by overt authority or covert sanctions. Comment: This is suggested by the observation that some people consider it none of their business to know or inquire how many houses or people there are in their village, since this is information relevant only to the authorities who impose and collect taxes; however, the potential universality of at least simple computation involved in buying and selling, and the existence of arithmetic games, suggests that it is something other than mathematics as such that is protected by authority or ceremonially restricted.

(6) More generally, the local culture may discourage the development of exploratory reasoning and scientific curiosity among children, substituting authoritative transmittal and a highly developed memory faculty for investigation and reason as the source of information. Comment: Apart from a detailed investigation of all factors, this would appear to be an exceedingly significant one; however, its impact on the individual can certainly be combatted much more readily in childhood than in later years (see 3 above). The local cultures do not seem to attempt to interfere seriously with what the school is trying to accomplish. The problem of the teacher as an adult participant in the local culture (and thus as the transmitting authority rather than the guide into independent reasoning) may, however, be crucial here.

(7) There may be elements in the structure of the student's home language which make it significantly difficult for him to conceptualize what he is taught in English. Comment: A detail of this type is well known: most Liberian students find it more difficult to understand "smaller than" than "larger than", and in at least

many of the indigenous languages of Liberia all such comparisons are reduced to the formula "larger than". The question is, how extensive is such linguistic interference? Is it substantial for even the most basic mathematical concepts and processes? An investigation of this question for only one of the languages of Liberia may be adequate; if one language -- say Kpelle -- provides no extensive evidence for such linguistic interference, then it is not likely that language is the culprit, for the speakers of this language are among those who have the difficulty which concerns us.

It is the purpose of this study to investigate only the last of these possible reasons for difficulty in learning mathematics. Preparatory to this, discussions were held with Dr. John Gay and Mr. John Wealar, and a preliminary statement was prepared by Dr. Gay concerning the logical formulation of quantitative propositions in English. It was recognized that such a logical formulation may be conditioned by the linguistic structure of English (and/or other languages in which the science of mathematics has been conveyed). If, however, it can be shown that quantitative propositions in Kpelle may be analyzed by reference to the same model, then it would follow that linguistic interference in this area of mathematical conceptualization is not significant. (It may also follow that a universal logical formulation of quantitative propositions is being approached; this, however, is beyond the scope of our immediate attention.)

A brief summary of the formulation of English quantitative propositions is as follows: a quantitative proposition includes, explicitly or implicitly, five components: content, form (~ presentation), aspect, measure, and enumerator. These components may be further described as follows:

(1) Content: the material or stuff referred to; e.g., water, stones, rice, money.

(2) Form: the particular shape or arrangement in which the content is observed; e.g., set, bundle, pile, bottle-full.

(3) Aspect: the type of quantity being measured; e.g., volume, number, length, weight, value.

(4) Measure: the unit of quantification; e.g., pint, yard, pound, item, cent.

(5) Enumerator: numbers and similar expressions; e.g., one, five, two hundred fifty, many.

(4-5 Portmanteau forms: heavy, long, far, much.)

Some illustrations of this formulation are as follows, first in chart form and then in corresponding sentence propositions:

	Content	Form	Aspect	Measure	Enumerator
1.	water	bottle	volume	pint	two
2.	cutlass	set	number	item	three
3.	cloth	strip	length	span	twenty
4.	load	set	weight	pound	forty
5.	money	currency	value	dollar	five
6.	box	set	weight	heavy	

1. This bottle of water is two pints in volume. (This bottle of water is two pints; there are two pints of water in this bottle; here's two pints of water.)

2. This set of cutlasses is three items in number. (These cutlasses are three in number; there are three cutlasses here.)

3. This strip of cloth is twenty spans in length. (This strip of cloth is twenty spans long.)

4. This set of load(s) is forty pounds in weight. (This load weighs forty pounds.)

5. This money, in the form of currency (not check), is five dollars in value. (This is five dollars.)

6. This set of box(es) is heavy in weight. (This box is heavy; these boxes are heavy.)

In performing mathematical operations, the numerical scale (the fifth component of the above model) is abstracted from the model as a whole, and it is assumed that the operations are valid for any specific manifestation of the entire model. For example, "2 + 2 = 4" abstracts the numerical scale from the model as a whole (or, reduces the model as a whole to the numerical scale or enumerator component). That is, "2 + 2 = 4" is a substitute for <u>any</u> full proposition of the following type: "A set of chairs two items in number added to a set of chairs two items in number equals a set of chairs four items in number" (i.e., "two chairs plus two chairs equals four chairs"). In this abstraction (or reduction), the numbers become names of abstractions rather than modifiers of measure nouns or (by apocopation of the logical formula) of content nouns.

The next step is to inquire whether quantitative propositions and mathematical operations as expressed in Kpelle may be analyzed in terms of the above formulations. It is not necessary that the full formulation of a quantitative proposition be expressed in Kpelle; after all, the English formulation "This bottle of water is two pints in volume" is not likely to be elicited from an unsophisticated informant, but is simply a way of combining in one formula all the relevant types of things that may be said about a quantified nominal. The formulation reflects more typical sentences like the following:

"This water is in a bottle."
"We would like to know the volume of this water."
"Let's measure this water in pints."
"I counted two pints of water."

If Kpelle has somewhat comparable expressions, reflecting terms in each of the five categories, related to each other in ways paralleling English usage, then Kpelle indeed quantifies observable phenomena in ways which present no obstacle to one who must learn the English structure of quantification. Of course, individual terms need not have precise equivalents in the two languages, as long as the categories of terms are parallel. Even in the examples above, the word "span" was used with a Kpelle measure for length of cloth in mind: the distance from finger-tip to finger-tip with the arms outstretched; this is not a precise measure by our standards, to be sure, but it is a usable one.

In the following paragraphs, each of the five components of the English-based formulation given above will be considered in order, and some typical examples of Kpelle usage given as illustrations.

1. Content terms are generally obvious, and present no particular problem. If mathematics were actually to be taught in Kpelle, a warning would be in order concerning the use of dependent nouns (e.g., kinship terms, body parts) as opposed to free nouns; to say "I have two feet" requires quite a different construction from saying "I have two chickens". Otherwise, it is not surprising to discover that nominals like the following can be quantified:

mɔlɔŋ	'rice'	wúru	'tree(s)'
pɛ́rɛ	'house(s)'	yá	'water'
kɔni	'stone(s)'	kâli	'hoe(s)'

A list of such content terms would, of course, be unrestricted

2. Form terms are probably much less varied than in English, but there are several clear examples. Some expressions for form are free nouns, but a few which are

particularly significant for mathematics are dependent nouns. Among free nouns, words for containers are especially useful for expressing form; e.g.,

kpôlo	'basket'	bôki	'bucket'
sane	'bottle'	leɤi	'pot'

That these may function as form words, and not exclusively as measures, is evident from sentences like:

gaménɛ puu náaŋ káa gbôlɔɪ ŋí sù. 'There are forty oranges in this basket.'

Here the measure is items (implied), and the aspect is number (implied). The content, oranges, merely has the form of being contained in a basket.

Dependent nouns expressing form may be illustrated by the following:

-kpulu	'(small) group, heap'
-kpuma	'bundle, package'
-seêi	'position, placement, set

The last of these deserves particular attention because of the distinctively mathematical usage of the English word 'set'. /-seêi/ is a verbal noun derived from the verb /see/, which has the intransitive meaning 'sit down' and the transitive meaning 'set down'. /núu seêi/ may be 'a place for a person to sit'; /í seêi káa sɛ́/ 'your sitting place is here' may be used in offering someone a seat: 'here's a place for you to sit'. By extension, /gâloŋ seêi/ can refer to 'the chief's headquarters'. /zeê lɛ́lɛɛ̂i/ 'his position is good' can, in context, be the equivalent of 'he is in good circumstances'. In the manipulation of small, moveable objects, however, /-seêi/ is used (without artificial elicitation) very much as the mathematician would use 'set': /kɔni saaba káa zeêi ŋí sù/ 'there are three stones in this "set" (of them).'

Words which in some combinations clearly express form are frequently used as second members of compounds, the first member in each case being a content word. Both free and dependent nouns appear in such compounds, some of which are:

núu-kpùlu	'a group of people'
mɔlɔŋ-kɔ̂ŋ	'a bundle ("neck") of rice'
mɔlɔŋ-kɔ̂ni-kɔ̀lɔŋ	'a stack of rice bundles'
seɣe-kpûu	'a ball of thread'
mɔlɔŋ-kâu	'a grain of rice'

It should be noted that 'compound' has a very specific definition in Kpelle linguistic structure; the tone of the second member of the compound, which is not ordinarily marked in Kpelle writing, is completely automatic: falling after mid, and low after other tones. In the third example above, the second member is /kɔni-kɔ̂lɔŋ/, itself a compound. The distinction between compound and phrase may be illustrated by comparing the second illustration above with /ŋ̀îlaĩ kɔ́ŋ/ 'the dog's neck'.

These compounds (perhaps in significant distinction from the most closely comparable English expressions) may have to be taken as simple content expressions rather than content-plus-form expressions. The English phrase "two bundles of rice" may be interpreted as "two items (aspect: number) of rice in the form of bundles"; but the Kpelle equivalent /mɔlɔŋ-kɔ̂ŋ feerɛ/ may rather be interpreted as "two items (aspect: number) of rice-bundles in the form of a set". In both English and Kpelle, to be sure, the assignment of a given term to a particular component of the logical formulation of quantification is not always determined by the identity of the term itself. A "bucket" may be content in one instance, form in another, and measure in still another. If there appears to be good reason (other than linguistic structure, which offers the suggestion) to consider the above Kpelle compounds as content expressions, then it would appear that the concept of "set" is even more basic in Kpelle than it is in English.

3. Aspect terms represent, in the English formulation, the highest level of abstraction in the entire discussion of quantification. Words like "number, weight, volume, length" are in quite a different area of vocabulary from "group, basket, pile" or "pound, inch, quart". The same is true in Kpelle; the typical aspect words are abstract nouns consisting of a stem plus /-là/, in which the stem is a verb which may also be used adjectivally or from which an adjectival may be derived. Semantically, these are nouns which one might well expect to be dependent nouns; however, they seem to be used primarily as free nouns. Some examples are:

kɛ́tɛ-là	'size'	(kɛ́tɛ: 'big, become big')
wíɛ-lâ	'weight'	(wíɛ: 'become heavy'; wíɛɛ 'heavy')
kôya-là	'length, distance'	(kôya: 'become long, far')
táma-là	'number'	(táma: 'become many'; támaa 'many')

It will be noted that it is always the maximum term ('big, heavy, long, many') that enters into these compounds, not the opposite minimum ('small, light, short, few'); in fact, some of the latter are expressed in Kpelle only as negatives of the maxima. Yet the abstracts do not refer to the maxima ('bigness, heaviness, multiplicity'), but are neutral in meaning; English 'length' and 'distance' are perfect parallels. Kpelle has no "neutral" terms other than these for 'size, weight, number'; but then, neither does English, any more than Kpelle, for linear measures.

There also appear to be a few instances of aspect expressions which are dependent nouns or noun phrases. The dependent compound /-su-kɛ̂ɔŋɔ/ is used for 'width', but perhaps also for other measurements in other contexts; it refers to the 'inside measured'.

4. Measure nouns are of two types. One type is a free noun (many of these are borrowed from English) indicating an external standard of measurement. These include:

pâi	'pint, salmon-tin (in rice, equals a pound)'
pâu	'pound (avoirdupois or monetary, equal to $4)'
kulûɒ	'five-gallon tin'
kâpa	'cent'
sêleɒ	'twenty cents (shilling)'
dâla	'dollar'

The forms /fóɒ/ 'five cents', /nɛ́i/ 'ten cents', and /ée-tì/ 'fifteen cents' are portmanteaus for measure and enumerator; they are not themselves enumerated. The compounds /fóɒ-kàu/ 'nickle' and /nɛ́i-kàu/ 'dime' are content expressions.

The second type of measure is a dependent noun used with reference to the measurer as possessor. Examples of this type are:

núu ɓwaɒ	'a person's outstretched-arm span' (or, ɓ̋waɒ 'my span', ɓ̀waɒ 'his span', í ɓwaɒ 'your span', or with any other appropriate possessor)
núu kpasa	'a person's arm-length (center of chest to finger tip)'
núu yée láa	'a person's hand laid out (thumb to index finger, or other form as demonstrated)'
núu kɔ́ɔ	'a person's foot (heel to toe)'

5. Enumerators include, of course, the entire numerical system of Kpelle, which has been outlined elsewhere. In addition, the following forms may function as enumerators; they are given with their grammatical classification:

tamaa 'many, much' (adjectival)
da / ta 'some, a certain' (demonstrative)
kélee 'all' (numeral)

The first of these may be a portmanteau for measure and enumerator when used with nouns which are not counted by item. Other such portmanteaus include quantitative adjectivals such as /wiɛ́ɛ/ 'heavy', much as in English.

Just as no measure is explicitly present in normal English sentences for "item", so Kpelle has no such term. English, however, does have the capacity for referring to items as "members of a set". In Kpelle, this is not true for all mixed sets. If a set includes no personal items, a member can be called /sɛŋ/ 'thing'. If all the non-personal items are animals, /sua/ 'animal' would undoubtedly be used to indicate members of the set. A set with all personal members is no problem either; /núu/ 'person' would indicate a member. But mixed personal - impersonal sets cannot be conveniently itemized in Kpelle.

The basic mathematical operations are performed most readily in Kpelle on the proto-mathematical level -- that is, with reference to a specific content rather than by abstracting the enumerator component. This distinction will first be illustrated with addition:

tɛ́ɛ feerɛ pɛlɛ́ɛ tɛ́ɛ saaɓa mà káa à tɛ́ɛ lɔ́ɔlu 'two chickens added onto (/-pɛlɛ́ɛ ...·mà/ three chickens is five chickens'

By using the "referred" form of the numeral, indicating "previous reference", the specific content noun can be eliminated in the second and third instances:

tɛ́ɛ feerɛ pɛlɛ́ɛ zaaɓa mà káa à ǹɔ́ɔlu 'two chickens added onto three (of them) is five (of them)'

This can be extended further to include the first instance of a numeral; however, a referent is always involved oven though not overtly expressed:

veerɛ pɛlɛ́ɛ zaaɛa mà káa à ŋ̀ɔ́ɔlu 'two (of them) added onto three (of them) is five (of them)'

Mathematics in the technical sense is present only if the enumerator is completely abstracted from the content. In Kpelle, true "mathematics" is present only if, in an operation such as the above, the numerals can be used in their stem forms rather than their "referred" forms. A real Kpelle mathematical formula would be:

feerɛ pɛlɛ́ɛ saaɛa mà káa à lɔ́ɔlu 'two added onto three is five; $2 + 3 = 5$'

Now, this type of formula is entirely acceptable to, and even welcomed by, informants who have been exposed to mathematics in school. Whether such a formula exists in a strictly pre-literate indigenous environment is another question. If it does, then the problem of teaching mathematical abstraction in education is certainly not linguistic; even if it does not, abstraction would not seem to be a very difficult step, since the word stems of numerals can be readily recognized by any speaker of the language.

There are two simple formulae for addition:

A pɛlɛ́ɛ B mà	'A added (<u>lit.</u> spread) onto B'
A púɔ B mà	'A poured onto B' (for non-itemzed masses or small items)

There are two simple formulae for subtraction:

A sɛyɛ́ɛ B mà	'A taken away from B'	(B - A)
A kuláa B mà	'A takn off from B'	(B - A)

These expressions, of course, parallel English "A from B"; "B minus A" is a new and strange inversion, as it probably is for English-speaking school children as well.

A formula for multiplication is:

A seêi B 'B sets of A'; e.g., tɛ́ɛ feerɛ seêi saaɓa káa à ŋ̀ɔ́ɔlu mɛi da 'three sets of two chickens is six of them'

A formula for division is:

A ɓá kɔ́lɛɛ à zeêi B 'A divided up into B sets of it'

In this case, the solution must be stated as a number per set. An illustration may be compared with a related proposition:

gwêi puu ɓá kɔ́lɛɛ à zeêi lɔ́ɔlu, zeêi tɔ̀nɔ káa à veerɛ 'ten bananas divided up into five sets of them, one set of them is two of them'

gwêi puu ɓá kɔ́lɛɛ núu lɔ́ɔlu mà, gáa à núu tɔ̀nɔ gwêi feerɛ 'ten bananas divided up on five persons, it is one person two bananas'

There are, as might be expected, many syntactically alternate ways of using these same elements, as there are in the English parallels, and other obvious ways of expressing arithmetic operations. Another form of multiplication, for example, is illustrated in this problem:

sáa náaŋ kɔ́ɔ káa à ɣɛɛlu 'the feet of four sheep are how many?'

All of this, however, merely adds confirmation to the existence of proto-mathematical operations with numbers of particular measures ("items" implied in all the above examples). It does not answer the question of whether abstract mathematics, using the numeral stems as names, exists or would be immediately intelligible.

The expression for equivalence in the above examples, A /káa à/ B, is the usual Kpelle expression for either equating one nominal with, or describing it as, another nominal. It is not of immediate importance here, but perhaps deserves mention, that the form of the expression is a singular imperative, "see A as B"; it has a special status in expressions of equivalence or description, however, since it can be used when talking to more than one person.

Approximation is expressed as /sòla/ A (northeastern dialects /kêlaa/ A), 'approximately A'. Precision is expressed as A /títi/ 'precisely A'. The function of approximation in computation has not as yet been investigated. Another expression related to equivalence is:

díkáa tonɛ́ɔ 'they are the same thing'

Although the immediate background (temporally) of the foregoing study includes some hours of work with Mr. John Wedar, a native of Kpaiye (extreme northeastern Kpelle in Liberia) and a college senior, much of the material was first recorded in 1946-48 and 1954-55 from less sophisticated informants (especially two at approximately fourth grade level) and by random observation of quantificational and computational phenomena among completely illiterate informants. For example, the borrowed words for measures are known to be universal in Liberian Kpelle society (in Guinea, of course, comparable borrowings from French are used). Another such measure, /âsa/ 'hour', is known from elicitation from informants who were in school; it is quite dubious whether the term means much of anything to illiterate members of the society. In any case, however, all the evidence points to the conclusion that the Kpelle language, and Kpelle culture in all its manifestations, possesses a framework of terminology and operations quite adequate as a background for the formal study of mathematics. A more detailed analysis will proceed from this preliminary investigation, and such analysis will provide further examples of specific difficulties (of the type of "smaller than") for the Kpelle student, as well as specific opportunities (of the type /zeêi/ for 'set of it') to build on the indigenous foundation. But it would appear that, in a very general way, the difficulties that Liberian students have with mathematics will largely have to be sought in areas other than that of language.

25 January, 1964