

~~1.2121~~  
it can be viewed as a non-countable mass.

1.212122 Thus what appear as members to one person may not appear as members to another.

In this example, the one-inch lengths would be members from one point of view, but not from the other.

1.212123 Thus the observer determines the nature of the field of attention, and there is no such thing as a field of attention independent of the observer.

1.2122 In order to test for the presentation-name of a set, the same open sentence can be used as in 1.12.

1.21221 A further test is the open sentence "this \_\_\_\_\_ of \_\_\_\_\_ is a set of \_\_\_\_\_," where the first blank contains the presentation-word and the second and third blanks contain the content-name.

1.21222 Thus the particular presentation-name used will be a special case of the presentation-name "set".

A bag of oranges is a set of oranges; a group of men is a set of men; a pile of stones is a set of stones.

1.213 We ask next of a countable field of attention the question of aspect, the way in which the presentation of the given content is considered.

1.21301 It is possible to consider a countable field of attention from some uncountable aspect.

It is possible to think about the weight of a bag of oranges.

1.21302 But such points of view are best considered as a sub-section of 1.22.

1.2131 The natural aspect under which to consider a given set of members counting is number.

1.21311 Number can thus at this preliminary stage be defined as the countable aspect of a set of members.

1.21312 We will find later that number is also relevant to non-countable sets, since it appears at the stage of value-name or number-name.

1.213121 This will require a modification and elaboration of the preliminary definition of number.

1.213122 However, this new definition will be really only an extension of the old definition to take it into account a new situation.

To be specific and thus to anticipate what is to come later, such numbers as the rational and real numbers must be invented to take care of the situation which arises with non-countable fields of attention.

1.2132 The open sentence used in 1.13 will demonstrate that number is a proper aspect-name for a set of members.

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- 1.21321 To demonstrate this further we can use the open sentence "I am thinking of the number of this set of \_\_\_\_\_," Where the blank gives the content-name.
- 1.21322 There are, however, no other aspect-names which are special cases of the aspect-names "number".
- 1.2133 Number is the natural aspect under which to consider the isolated, discontinuous individuals in a set.
- 1.21331 Number is the property which a set has when it is considered according to its discreteness.
- 1.21332 Other properties of a presentation consider it apart from any possible discreteness it may display.
- 1.213321 These other properties are thus to be considered as uncountable aspects.
- Weight, as applied to a set of oranges does not take into account the discreteness of the oranges, and could equally apply to the same oranges mashed into a pulp.
- 1.213322 This split between number and all other aspects is the most basic feature of the split between countable and uncountable fields of attention.
- 1.214 We ask next of a countable field of attention the question of measure, the way in which the given aspect of the presentation of the given content is valued.
- 1.21401 It is possible to consider a countable field of attention from many aspects, and thus to impose uncountable measures upon it.
- If we are thinking of \_\_\_\_\_ a bag of oranges, we must measure that weight in a unit such as pounds.
- 1.21402 But such points of view are best considered as a sub-section of 1.22.
- 1.2141 There are several possible measures which can be imposed on the countable aspect which is number.
- We can, for instance, consider the number of a group of people in terms of individuals or pairs or foursomes.
- 1.21411 However, each of these is a multiple of the basic measure in terms of individuals.
- 1.21412 Thus we will take as the basic measure of number the individual, which can also be called the item.
- 1.214121 The item is one single member of a discrete set, considered apart from its content.
- Thus, considered from this abstract point of view, the item for the set of oranges is the same as the item for the set of people.
- 1.214122 The content enters at an earlier stage, and thus need no longer be considered at this point.

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- 1.214123 The item is the measure of the unit set, and allows us to compare the measure of the given presentation, or set, with the measure of the unit set.
- 1.214124 The comparison is done in terms of number, which is the aspect.
- 1.214125 Thus a countable field of attention can be defined as one which can be viewed as a combination of discrete unit fields of attention.
- Anything considered from this point of view is countable, even a ruler which is looked on as series of inch-lengths placed end-to-end.
- 1.214126 The normal way of looking at certain fields of attention is as countable, terms of items, as is shown by the plural in the content-name.
- Thus the name "oranges" with its plural form shows that a set of oranges is looked at as consisting of a collection of single items.
- 1.21413 Other measures can be constructed by putting together items into larger measures.
- 1.2142 The open sentence used in 1.14 will demonstrate that item is a proper measure-name for a set of members.
- 1.21421 Likewise each of the derived measure-names will fit this open sentence.
- 1.21422 None of the measure-names derived from uncountable aspects will be special cases of this measure-name.
- If we think of the number of inches in a piece of string, we are thinking of the string as uncountable, in terms of length.
- 1.2143 Item is the natural measure which should be imposed on the isolated, discontinuous individuals in a set.
- 1.21431 Item is the measure imposed on a set which it is considered according to its discreteness.
- 1.21432 Other measures of a presentation consider it under some aspect which is not related to its discreteness.
- 1.214321 Thus these other measures are considered to be uncountable measures.
- The pounds by which a set of oranges are measured do not take into account the discreteness of the oranges, and could equally apply to the same oranges mashed into a pulp.
- 1.214322 This split between item and its multiples and all other measures is basic to the split between countable and uncountable fields of attention.
- 1.215 We ask last of a countable field of attention the question of value, the way in which the given measure of the aspect of the presentation of the content is actually numbered, the range of possibilities of number.

- 1.2151 There are several possible number systems which can be used to make specific the countable measure, which is formed from item and its multiples.
- 1.21511 These number scales are those mentioned in 1.15, which are of complete generality and apply equally to all fields of attention.
- 1.21512 In particular, it is possible to use the range none-some-all, or the range of ordinary counting numbers.
- 1.21511 The ordinary counting numbers give the most natural, and yet precise, values to the measure by items.
- 1.21512 These counting numbers enable us to give a precise comparison between the given presentation and the standard unit presentation.
- 1.21513 Thus every set, which is a presentation of a countable field of attention, has as a property number, which is measured by items, and valued in counting numbers.

Thus we can speak of a set of seven oranges.

- 1.21513 The counting numbers apply equally to the measure by pairs or foursomes or any other multiple of unit items.
- 1.2152 The open sentence used in 1.15 will show the names of these numbers to be value-names for the set of members considered from the aspect of number and measured in items or some multiple of items.
- 1.21521 The range all-some-none will fit this open sentence.
- 1.21522 The range of ordinary counting numbers will fit this open sentence.
- 1.2153 Other numbers will be found which fit the countable fields of attention only indirectly.

For instance, we can use special fractions, namely, those which are equivalent to whole numbers, as numbers to fit countable fields of attention. But this is clearly artificial. The real use of fractions will be seen under 1.22 to be in connection with uncountable fields of attention.

- 1.21531 This split between counting numbers and other extended number systems will be found to be the split between countable and uncountable fields of attention.
- 1.21532 These other number systems will be considered uncountable value scales.
- 1.2154 The counting numbers thus number discrete sets of points.
- 1.21541 A line consisting of a finite number of discrete points will have a number, but not a length, since its measure, in the uncountable sense will be 0, and its number in a countable sense will be finite.
- 1.21542 A line consisting of all points in a neighborhood will have a length, but not a number, since its measure in an uncountable sense will be finite, and its number in a countable sense will be infinite.

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1.22 We consider next uncountable fields of attention.

1.221 We ask first of an uncountable field the question of content.

1.2211 The basic fact about an uncountable field of attention is that the content-work is indifferent either to singularity or plurality.

For instance, rice and water are neither singular nor plural, but are simply *uncountable*, whereas oranges and people are countable, since the singularity or the plurality are indicated in the content-name.

1.22111 Thus we do not learn anything about the value of the measure of the uncountable field of attention from the way in which its content-name appears in the description.

1.22112 The content-names in general can be found by using the open sentence of 1.11.

1.2212 A term which will cover all content-names for uncountable fields of attention is "material".

It I attend to an uncountable field of attention, I can consider whatever I find in it, whether rice or red/<sup>or</sup>water, as material.

1.22121 Material is not inherently individual or isolable.

1.22122 Thus material is essentially continuous.

1.221221 This continuity need not be physical, spatial continuity, but can be some other type of continuity.

1.221222 The continuity is a question of the way in which the field of attention is considered, since in one way the field of attention may be continuous, and in another way discontinuous.

As in 1.211222, a group of people is countable with respect to number, and thus is discontinuous. However, with respect to weight it is continuous, since the physical separation between the persons does not mean a necessary physical separation on the level of weight. From this point of view, the physical separation is simply not attended to. Likewise, it is possible to view a ruler either as a continuous length, or as a discrete collection inch-lengths placed end to end.

1.221223 We remember from 1.211223 that the field of attention is the content as it is presented, considered from an aspect, measured by a measure, and numbered by value.

1.221224 The continuity of the field of attention is what makes it impossible for the field to be counted, since there are no items to be counted, unless arbitrary sub-divisions are made in the field, according to the units.

1.22123 Thus a test for material is an open sentence of the type "\_\_\_\_\_ is the material in the field of attention," where the blank gives the content-work.

For instance, we can say that "rice is the material in the field of attention" or "sound is the material in the field of attention".

- 1.221231 This test will *succeed* for uncountable fields of attention.
- 1.221232 This test will *fail* for countable fields of attention.
- 1.222 We ask next of an uncountable field of attention the question of presentation, the way the content is presented.

1.22201 It is difficult to find a general way of describing uncountable fields of attention.

Such fields of attention vary widely, from a pool of water, to a note of music to a moment of thought, and are not so easily characterized as are sets of countable members.

1.22202 Thus any description which is obtained must necessarily be somewhat abstract, and alien to ordinary speech.

1.2221. Continuum is a ; general presentation-word for the way the content of an uncountable field of attention is formed.

We can speak of a continuum of rice, or a continuum of milk, or a continuum of yellow, but not of a continuum of oranges, or a continuum of people, or a continuum of ideas. Or, if we do consider uncountable things in terms of continuum, we are considering them in an uncountable way.

1.22211 Thus we can define a continuum as presentation of material.

1.222111 This is a definition in terms of previously defined terms, and proceeds from the more general to the more specific.

1.222112 The term continuum is the most presentation-name for a presentation of an uncountable field of attention, and is applicable to any presentation of an uncountable field of attention.

1.222113 The term continuum is not applicable to any countable field of attention, unless it is considered from some uncountable point of view, in which case it is in fact an uncountable field of attention.

We must remark that we can think of oranges in terms of their weight, in which case we are considering the oranges from a continuous, thus uncountable aspect.

1.22212 We can also see that the continuum is a presentation of material considered without any isolated segments or discontinuous individuals.

1.222121 This is a question of the way the field of attention is viewed.

1.222122 Thus what appears as material to one person may appear otherwise to another.

To quote the example under 1.212121 and 1.212122, we see that a

ruler can be considered as an uncountable mass, as well as a set of one-inch lengths set end to end.

- 1.222123 Thus once again we note that the observer determines the nature of the field of attention, and that there is no such thing as a field of attention independent of the observer.
- 1.2222 In order to test for the presentation-name of a continuum, the same open sentence can be used as in 1.12.
- 1.22221 A further test is the open sentence "this \_\_\_\_\_ of \_\_\_\_\_ is a continuum of \_\_\_\_\_," where the first blank contains the presentation-word, and the second and third blanks contain the content-name.
- 1.22222 Thus the particular presentation-name will be a special case of the presentation-name "continuum".

A pool of water is a continuum of water; a breath of fresh air is a continuum of fresh air; a bag of rice is a continuum of rice.

1.223 We ask next of an uncountable field of attention the question of aspect, the way in which the presentation of the given content is considered.

1.22301 It is possible to consider an uncountable field of attention from a countable aspect.

It is possible, for instance, to think about the number of grains in a bag of rice, or the number of molecules in a bottle of water.

1.22302 But in this way the field of attention is actually then countable, and the content and presentation names should be altered.

1.22303 It is clear that a continuum, from the point of view of language, is what we attend to and describe by our language, but not necessarily what the thing is in itself, which is in fact unimportant and irrelevant and even undiscussable.

1.2231 There is no single natural aspect under which to consider a given continuum of material, other than quality.

Obviously we can think of a pool of water in terms of weight, volume, temperature, color, usefulness, and so forth.

1.22311 There is a certain uniformity among all these aspects, in that they all are based on the continuity displayed by the material, and can be called qualities.

1.223111 Thus all these aspects are mass properties or properties of the material in the large, not properties which depend on any essential discontinuity in the content.

1.223112 It is true that number is also a property depending on the whole of the given presentation of content, but it also depends on the discontinuity of the content, and thus is best called quantity instead of quality.

1.223113 These aspects, which we call qualities, can be enumerated, as we will see under 1.224 and 1.225, but this numeration has a special character.

1.223114 The distinction made here, therefore, between quality and quantity, is a fundamental distinction, and corresponds to that between countable and uncountable.

This is the traditional distinction of the logical categories of Aristotle and others. It is here seen in a clearer light, and understood in terms of the total picture of the description of fields of attention. The categories can only be basic divisions in the way of describing the content of a field of attention, and thus this analysis gives the best description of the categories, one which is guaranteed to be complete.

1.2232 The open sentence used in 1.13 will demonstrate that quality is proper aspect-name for a continuum of material.

1.22321 To demonstrate this further we can use the open sentence, "I am thinking of a quality of this continuum of \_\_\_\_\_," where the blank gives the content-name.

1.22322 There are many aspect-names which are special cases of the aspect-name "quality" as we have seen above.

1.2233 Quality is the natural aspect under which to consider the continuous, undivided material in a continuum.

1.22331 Quality is the property which a continuum has when it is considered according to its continuousness.

1.22332 The only other property of a presentation apart from uncountable aspects, or qualities, is number, or quantity.

1.223321 This other property is thus the countable aspect of a presentation.

The number of a set of oranges, just as the number of inches in a ruler, takes into account discreteness, but not any quality.

1.223322 Thus once again we see that this split between number and all other aspects is the most basic feature of the split between countable and uncountable fields of attention.

1.224 We ask next of an uncountable field of attention the question of measure, the way in which the given aspect of the presentation of the given content is valued.

1.22401 It is possible to consider an uncountable field of attention from a countable aspects, thus considering it, in effect, as a countable field of attention.

We can think of a bottle of water in terms of the number of molecules in it, thus converting it into a set of molecules of water.

1.22402 But such points of view are best considered as a sub-section of 1.21.



1.2241 There are several possible measures which can be imposed on each quality, or uncountable aspect, by which we may consider a continuum of material.

We can measure the length of a piece of string in terms of inches or feet or centimeters. We can measure the beauty of a painting in terms of personal emotion or critical acclaim.

1.22411 Each of these basic measures is in terms of a standard unit of measure.

1.224111 The unit measure is an arbitrarily determined field of attention, which is compared with the given field of attention, and which has the same content, the same presentation, the same aspect, the same measure, but a different value.

For instance, the inch of string is the same as the piece of string in everything except value.

1.224112 There can be many standard units of measure, since each of them is chosen purely arbitrarily from the continuum which is viewed under the given quality.

Thus the inch and the centimeter are incommensurable lengths, since they are chosen arbitrarily. This could not happen, of course, with countable measures, since they must all be related to the item.

1.22412 The basic measure is then used as a standard of comparison, by which the given presentation can be given a number in terms of the standard.

1.2242 The open sentence used in 1.14 will demonstrate that unit is a proper measure-name for a given quality of a continuum of material.

1.22421 Likewise each of the special measure-names will fit this open sentence.

1.22422 None of the measure-names related to the countable aspect of quantity will be special cases of this measure-name.

We cannot think of a piece of string as being measured in terms, whereas it can be measured in units.

1.2243 Unit is the natural measure which should be imposed on the continuous, non-isolated material in a continuum.

1.22431 Unit is the measure imposed on a continuum when it is considered according to its continuity, rather than according to any type of discreteness.

1.22432 Thus it is distinguished from the basic countable measure, which is in terms of items, or multiples thereof.

1.22433 Thus we see once again that this split between unit, which is chosen from a continuum arbitrarily, and item, which is a natural measure of discreteness, is fundamental to the split between countable and uncountable fields of attention.

1.225 We ask last of an uncountable field of attention the question of value, the way in which the given measure of the aspect of the presentation of the content is actually numbered, the range of possibilities of number.

1.22501 The most notable fact by way of introduction is that the unit of measure and the given presentation do not have to be commensurate in terms of counting numbers.

For instance, there is no reason why a piece of string has to be an integral number of inches in length. There is no reason why an emission of sound has to be an integral number of cycles per second in frequency.

1.22502 This is in sharp contrast with the countable field of attention where the given field must be an integral number of items in value.

1.2251 Thus the number scales applied to units of measure must be more complex than those of the countable fields of attention.

1.22511 It is, of course, possible that the simple value-systems, such as all-some-ness, and the counting numbers, can apply to an uncountable field of attention.

It is possible to have a piece of string three inches in length, or a painting exactly as beautiful as the standard painting.

1.22511' In this case, the ordinary counting numbers, which are the countable value-scale, apply to the uncountable field of attention, which is thus considered as if it were in effect countable.

To measure the piece of string as exactly three inches in length is to consider it as a set of three inch-lengths.

1.225112 In this case, the analysis under 1.213, 1.214, and 1.215 apply to the uncountable field of attention considered as if it were countable.

1.225113 Thus the instruction to count unit measures in terms of ordinary counting numbers is to consider the field of attention as composed of those unit measures combined together to make the field of attention.

1.225114 Thus the behavior in this case for the uncountable case is a repetition of the behavior previously described for the countable case.

1.22512 It may be, of course, that the counting numbers do not apply in which case other value-systems are necessary, in order to make the given presentation and the standard presentation commensurate.

1.225121 One such value system is that of ordinary fractions, in which case the given presentation, is, in effect, compared with standard portions of the standard unit measure.

For instance we might say that a piece of string is  $1 \frac{5}{8}$  inches in length, by which we mean that we measure it in terms of  $1/8$ -inch units, and we find that there are 13 of them.

1.2251211 This value system is just an extension of the previous value system, since it uses a smaller unit measure, a multiple of which is the original unit measure.

1.2251212 Thus once again the analysis given for countable fields of attention, considering the aspect as number, the measure as item and the value system as counting numbers, is applicable.

1.22522 A more complex case is where no fraction of the standard unit measure is commensurate with the given presentation.

For instance, the circumference of a circle is  $\pi$  times the diameter of the circle, and  $\pi$  is not a fraction. Likewise the diagonal of a unit square is the square root of two, which is not a fraction.

1.225221 In this case the irrational numbers are introduced as a more complex value-system, including more numbers than the counting numbers or even the fractions.

1.225122 This value-system behaves as the limiting case of the fractions, and is the means by which every point on an ordered continuum receives a number from the value system.

1.225123 Another complex case is where the field of attention consists not of positive content, but of the lack of content.

I can attend to a debt which I owe to another man. This debt might be countable, as a debt of 10 oranges, or might be non-countable, as a debt of rice.

1.2251231 In the case the deficiency is countable, I can refer to it with the so-called negative <sup>integers</sup> fractions or the negative irrational numbers.

1.2251232 In case the deficiency is uncountable, I can refer to it with the negative fractions or the negative irrational numbers.

1.2252 The open sentence used in 1.15 will show that all these different value systems provide value-names for the continuum of material considered from the aspect of quality and measured in units.

1.22521 The range all-some-none will fit this open sentence.

1.22522 The range of ordinary counting numbers will fit this open sentence.

1.22523 The range of fractions will fit this open sentence.

1.22524 The range of irrational numbers will fit this open sentence.

1.22525 The range of negative numbers will fit this open sentence.

1.22526 There are other value-systems, which we need not consider here, which will fit this open sentence.

- 1.22527 Once again, there is a fundamental split here between counting-type numbers (including counting numbers, integers, and fractions) and uncountable-type numbers (including primarily the irrational numbers).
- 1.22531 We see that the one type is countable and is basically related to countable fields of attention.
- 1.22532 We see that the other type is uncountable, and is basically related to uncountable fields of attention.
- 1.2254 Thus we see that the uncountable numbers, as well as the countable numbers, considered as special or limiting cases of the uncountable numbers, measure continuous sets of points.
- 1.22541 A line segment consisting of a finite number of discrete points will not have a length, and thus will have measure zero in this sense, even though its countable number will be finite and non-zero.
- 1.22542 A line segment containing all points in a neighborhood will thus have a length, which can be given a finite uncountable number, but it will not have a countable number in the proper and original sense, since its measure will be finite, but its number of points will be infinite.
- 1.22543 Thus, even though the counting numbers can be embedded in the uncountable numbers as limiting or special cases, there is basically a fundamental split between the two kinds of numbers, just as there is between the two fields of attention.
- 1.3 We now consider the field of attention restricted so that our discussion is at the third level of generality.
- 1.301 We have thus far considered fields of attention of absolute generality, and then considered fields of attention which were either countable or uncountable, and have in every case given a thorough analysis of each of the five content-centered features of describing these fields of attention.
- 1.302 We must thus now consider the basic splits of types of countable field and types of uncountable field, but we will not make a complete analysis of language behavior.
- 1.31 We will consider first countable fields of attention, and consider the types of such fields of attention.
- 1.311 The first type of countable field of attention consists of physical objects, encountered in the world of space, which may be animate or inanimate.
- 1.3111 Physical objects occur as members of collections, and their number-aspect is measured by itms.
- 1.3112 This type of field of attention corresponds to extended mass, as the stuff out of which it is made
- We can see this in many examples. Consider trees, which are made out of wood, as the stuff or material. The trees are countable, physical objects, made out of the uncountable material, which is wood.
- 1.312 The second type of countable field of attention consists of completed events, which are encountered in the world of time.

We can speak in this connection of the wars and revolutions which took place in the past, which events are clearly countable.

1.3121 These events can also be considered to be members of sequences, and once again their number-aspect is measured by items.

1.3122 This type of field of attention corresponds to activity, as the stuff out of which it is made.

Thus a war is made out of fighting, which is itself uncountable, although the wars are countable.

1.313 The third type of countable field of attention consists of propositions or ideas, which are encountered in the world of mind.

We can think of the proposition "2 plus 2 equals 4", which is a single, countable proposition in the realm of mind.

1.3131 These propositions can be considered to be members of relations, and they are measured, according to their number-aspect, by items, and are joined together by logical procedures.

1.3132 This type of field of attention corresponds to quality or aspect, as the stuff out of which the propositions are predicted.

Truth is a quality or attribute, and so is fourness, and the proposition "2 plus 2 equals 4" is composed out of attributes of this type.

1.32 We will consider next uncountable fields of attention, and consider the types of such fields of attention.

1.321 The first type of uncountable field of attention consists of extended mass encountered in the world of space.

1.3211 Extended mass occurs as a continuum of matter, and has many aspects, which can be measured according to units.

1.3212 This type of field of attention corresponds to physical objects, which are integrations into simultaneous and completed whole of the mass of material.

We thus see that the integration of a mass of wood is a tree, or the integration of a mass of lumber is a house.

1.32121 We saw above under 1.225 that uncountable number applies to a continuum of material, measuring it in comparison to a standard, and identifying the measure of the infinite point set.

1.32122 The integrated material then becomes a discrete set and is numbered as a set of members according to items.

1.32 The second type of uncountable field of attention consists of activity, encountered in the world of time.

1.321 Activity occurs as a continuum of energy, and has many aspects, which can be measured according to units.

1.3222 This type of field of attention corresponds to completed events, which are integrations into a simultaneous and completed whole of the mass of energy.

We thus see that the integration of a mass of fighting is a war, or the integration of a mass of electric voltage is a shock.

1.3221 Once again we see that uncountable numbers apply to the continuum of energy, measuring it in comparison to a standard, and identifying the measure of the infinite point set.

1.3222 The integrated material becomes a discrete point set of events, and is numbered as a set of members according to items.

1.323 The third type of uncountable field of attention consists of quality, encountered in the world of mind.

1.3231 Quality occurs as a continuum of abstraction, and has many aspects, which can be measured according to units.

For instance, color can be considered according to intensity or hue.

1.3232 This type of field of attention corresponds to completed propositions, which are integrations into predictions of the mass of a quality.

It is an integration of yellow to say "This flower is yellow", since it applies the quality to a completed and unified whole.

1.3231 In this case also we see that uncountable numbers apply to the continuum of the property or quality, measuring it in comparison to a standard, and starting the measure of the infinite point set which is the quality.

1.3232 The integrated quality becomes a discrete proposition, and is numbered as a set of members by item.

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